

Second Mid Exam for The Second Semester 2024

الإختبار الفصلي الثاني للفصل الدراسي الثاني من العام الجامعي ١٤٤٥

								الرقم الجامعي Student's ID		اسم الطالب / ة Student's Name	
		رقم القاعة Room No				الشعبة Group		Math-211		رمز المقرر Course Code	
								تفاضل وتكامل ١ Calculus1		اسم المقرر Course Name	
إلى To		من From		وقت الاختبار Exam Duration		24/01/2024		التاريخ Date		Wednesday	
03:00 p.m		02:00 p.m								اليوم Day	
3				عدد صفحات الإختبار بدون الغلاف Number of Pages Without Cover Page						رقم التسلسل Serial Number	

Domains Percentages (%)				Second Mid Exam			
Learning Domains	Percentage in the course	Percentage in the Second Mid Exam	Marks in the Second Mid Exam	Question No	(CLOs)	Mark	Student Mark
Knowledge & Understanding	30	30%	6	1	K&U (CLO 1.1)	2	
					K&U (CLO 1.2)	2	
					K&U (CLO 1.3)	2	
Skills	60	70%	14	2	S (CLO 2.1)	4	
					S (CLO 2.2)	3	
Values	10			3	S (CLO 2.3)	3	
					S (CLO 2.4)	4	
Total	100	100%	20	Total		20	

Course Instructor	_____	Reviewer	_____
Signature	_____	Signature	_____

Answer the following questions.

(K1,K2,K3)

(Marks: 6).

Question1.

K1(1-2),K2(3-4)

(Marks: 4).

(1) Choose the correct answer.

Solution:

(1) $\lim_{x \rightarrow \infty} \frac{2x^2 + 2x + 3}{10x^2 - x + 1}$

(A) 10

(B) $\frac{1}{5}$

(C) $\frac{10}{2}$

(D) 2

(2) If $x < 0$, $\lim_{x \rightarrow 0^-} \frac{-x}{|x|} =$

(A) 1

(B) 0

(C) -1

(D) Does not exists

(3) $\frac{d}{dx}[\pi^2] =$

(A) 2

(B) π

(C) 2π

(D) 0

(4) $\lim_{x \rightarrow \infty} e^{-x}$

(A) ∞

(B) 1

(C) 0

(D) $-\infty$

K3

(2 marks)

(2) Determine the value of x that make the given functions are discontinuous.

Solution:

(a) $\frac{2x}{2x^2 - x}$ it's discontinuous at $x = \frac{1}{2}$, since it's not defined at $x = \frac{1}{2}$

mark 1

(b) $\frac{1}{x^2 + 2x}$ it's discontinuous at $x = 0, -2$, since it's not defined at $x = 0, -2$

mark 1

Question2.

(S1,S2. 7 marks)

(1) Find the equations of the tangent line to the curve $f(x) = 3x^2 + 2x - 5$ at the point $(0, 1)$.

Solution:

(Marks: 2).

$$f' = 6x + 2, \quad f' \Big|_{x=0} = 6(0) + 2 = 2 = m$$

mark 1

$$y - 1 = 2(x - 0) \implies y = 2x + 1.$$

mark 1

(2) Show that $\lim_{x \rightarrow 0} x \cos\left(\frac{1}{x}\right) = 0$.

(Marks: 2).

Solution:

Consider $-1 \leq \cos x \leq 1$

$$\implies -1 \leq \cos\left(\frac{1}{x}\right) \leq 1$$

mark 0.5

$$\implies -x \leq x \cos\left(\frac{1}{x}\right) \leq x$$

mark 0.5

Now, $\lim_{x \rightarrow 0} (-x) = 0$, $\lim_{x \rightarrow 0} (x) = 0$

mark 0.5

Therefore, $\lim_{x \rightarrow 0} x \cos\left(\frac{1}{x}\right) = 0$

mark 0.5

S2

(3 marks)

(3) Show that the given function is continuous at 2.

$$f(x) = \begin{cases} \frac{\sqrt{x+2}-2}{x-2} & \text{if } x \neq 2 \\ \frac{1}{4} & \text{if } x = 2 \end{cases}$$

Solution:

★ $f(2) = \frac{1}{4}$ is defined.

mark 1

★ $\lim_{x \rightarrow 2} \frac{\sqrt{x+2}-2}{x-2} = \frac{0}{0}$.

$$\implies \lim_{x \rightarrow 2} \frac{\sqrt{x+2}-2}{x-2} = \lim_{x \rightarrow 3} \frac{(\sqrt{x+2}-2) \cdot (\sqrt{x+2}+2)}{(x-2) \cdot (\sqrt{x+2}+2)} = \lim_{x \rightarrow 2} \frac{x+2-4}{(x-2)\sqrt{x+2}+2}$$

$$= \lim_{x \rightarrow 2} \frac{x-2}{(x-2)(\sqrt{x+2}+2)} = \lim_{x \rightarrow 2} \frac{1}{\sqrt{x+2}+2} = \frac{1}{4} \text{ exist.}$$

mark 1.5

★ Therefore $f(2) = \lim_{x \rightarrow 2} f(x)$.

mark 0.5

$\therefore f$ is continuous at $x = 2$.

Question3.

S3, S4. 7 marks.

(1) Find the derivative of $f(x) = x^2 - x$ by the definition of derivative.

(Marks: 3).

Solution:

$$f(a+h) = (a+h)^2 - (a+h) = a^2 + 2ah + h^2 - a - h, \quad f(a) = a^2 - a$$

mark 1

$$\begin{aligned} \star \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} &= \lim_{h \rightarrow 0} \frac{\cancel{a^2} + 2ah + \cancel{h^2} - \cancel{a} - \cancel{h} - \cancel{a^2} + \cancel{a}}{h} = \lim_{h \rightarrow 0} \frac{\cancel{K}(2a + h - 1)}{\cancel{K}} \\ &= \lim_{h \rightarrow 0} (2a + h - 1) = 2a - 1. \end{aligned}$$

mark 1

S4

(4 marks)

(2) Evaluate $\lim_{x \rightarrow 0} \sin^{-1} \left(\frac{\sqrt{x} + 1}{x + 2} \right)$

(Marks: 2).

Solution:

$$\lim_{x \rightarrow 0} \frac{\sqrt{x} + 1}{x + 2} = \frac{1}{2} \Rightarrow \sin^{-1} \left(\frac{1}{2} \right)$$

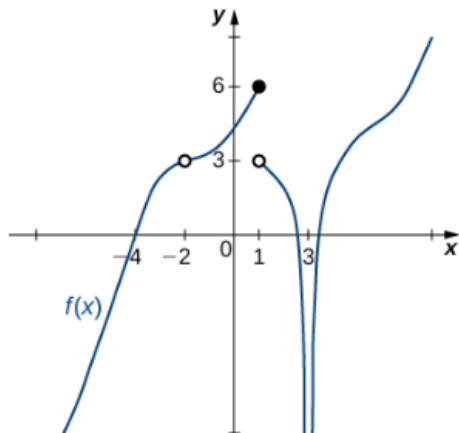
mark 1

$$\text{Let } \Theta = \sin^{-1} \left(\frac{1}{2} \right) \Rightarrow \sin \Theta = \frac{1}{2} \Rightarrow \Theta = \frac{\pi}{6}$$

mark 1

(3) From the graph of $f(x)$, evaluate $\lim_{x \rightarrow -2^+} f(x)$, $\lim_{x \rightarrow -2^-} f(x)$, $\lim_{x \rightarrow -2} f(x)$, and $f(-2)$

(Marks: 2).



Solution: $\lim_{x \rightarrow -2^+} f(x) = 3.$

mark 0.5

$$\lim_{x \rightarrow -2^-} f(x) = 3.$$

mark 0.5

$$\lim_{x \rightarrow -2} f(x) = 3.$$

mark 0.5

$$f(-2) \text{ undefined.}$$

mark 0.5

GOOD LUCK