Assignment-2
Calculus 1 (Math-105)
First Semester (2024-2025)

Q.1 Find y' of

(i) $y = x^3 - 5x^{\frac{1}{2}} + \frac{2}{x} + 9$ **(ii)** $(x^4 - 2x)(2x - 5)$ **(iii)** $\frac{(x^2-x+7)}{(3x-2)}$

(iv) $y = x^2 logx - cosx - 6x^3 + \frac{1}{x^2}$ **(v)** $y = \frac{3+sinx}{(2+x)}$ **(vi)** $y = \frac{x^2+x}{sinx+2cosx}$

Q.2 Find $\frac{dy}{dx}$ of (i) $y = \sin 3x^2$ **(ii)** $y = \log(1 + x + x^3)$ **(ii)** $y = e^{x^2}$

Q.3 Use implicit differentiation to find $\frac{dy}{dx}$, if

(i) $y^2 - 3sinx = x$ **(ii)** $2x^2 - 3cosy = 4$

Q.4 Find the interval on which

(i) $f(x) = 4x^2 - 11x + 5$ **(ii)** $f(x) = x^2 - x + 8$ **(iii)** $f(x) = 5 - 4x - x^2$ is increasing and decreasing

Q.5 Find the interval on which the following f(x) is concave up and concave down

(i) $f(x) = -3x^2 + 5x + 5$ **(ii)** $f(x) = x^3 - 7x^2 + 8$

Q.6 Find all the critical points of

(i) $f(x) = x^3 - 4x + 5$ **(ii)** $f(x) = 2x^3 - 15x^2 + 24x$ **(iii)** $f(x) = x^2 - 3x + 4$

Q.7 Find relative extrema of the function on the interval of

$f(x) = 4x^3 - 12x^2 + 36x$

Q.8 Find the two x- intercepts of the function $f(x) = x^2 + x - 2$ and confirm that $f'(c) = 0$ at some point c between these intercepts.

Q**.9 Verify that the hypotheses of Rolle's theorem are satisfied for** $f(x) = x^2 - 5x + 6$ on the interval [2,3] and find all the values of c in that interval that satisfy the conclusion of the theorem.

Q.10 Integrate the following:

(i) $\int (2x^4 - 4x^3 + x^{\frac{1}{2}} + \frac{2}{x} - 4)\, dx$ **(ii)** $\int cos(x^2 - 7)2x\, dx$ **(iii)** $\int sin^6 x\, cosx\, dx$

Q.11 Integrate the following:

(i) $\int_0^2 (x + 4x - 2x^3)\, dx$ **(ii)** $\int_0^{\frac{\pi}{6}} \frac{sinx}{4}\, dx$ **(iii)** $\int_0^3 (5x - 3)^3\, dx$ (iv) $\int_{\frac{\pi}{2}}^{\pi} cos2x\, dx$

السؤال الأول:

(i) $y = x^3 - 5x^{\frac{1}{2}} + \frac{2}{x} + 9$

$y' = 3x^2 - \frac{5}{2}x^{-\frac{1}{2}} - 2x^{-2} = 3x^2 - \frac{5}{2\sqrt{x}} - \frac{2}{x^2}$

(ii) $(x^4 - 2x)(2x-5)$

ضرب مشتقات

1. $x^4 - 2x \longrightarrow 4x^3 - 2$
2. $2x - 5 \longrightarrow 2$

$(4x^3 - 2)(2x-5) + 2(x^4 - 2x)$

$8x^4 - 20x^3 - 4x + 10 + 2x^4 - 4x$

$= 10x^4 - 20x^3 - 8x + 10$

(iii) $\frac{x^2 - x + 7}{3x - 2}$

قسمة مشتقات

1. $x^2 - x + 7 \longrightarrow 2x - 1$
2. $3x - 2 \longrightarrow 3$

$\frac{(2x-1)(3x-2) - (3)(x^2 - x + 7)}{(3x-2)^2}$

$\frac{6x^2 - 4x - 3x + 2 - 3x^2 + 3x - 21}{(3x-2)^2}$

$\frac{3x^2 - 4x - 19}{(3x-2)^2}$

(iv) $y = x^2 \log x - \cos x - 6x^3 + \frac{1}{x^2}$

$y' = 2x \log x + \frac{x}{\ln 10} + \sin x - 18x^2 - \frac{2}{x^3}$

(v) $y = \frac{3 + \sin x}{(2+x)}$

قسمة مشتقات

1. $3 + \sin x \longrightarrow \cos x$
2. $2 + x \longrightarrow 1$

$\frac{(2+x)\cdot \cos x - (1)(3 + \sin x)}{(2+x)^2}$

$\frac{2\cos x + x\cos x - 3 - \sin x}{(2+x)^2}$

(vi) $y = \dfrac{x^2 + x}{\sin x + 2\cos x}$

قسمة مشتقات

1 $x^2+x \longrightarrow 2x+1$

2 $\sin x + 2\cos x \longrightarrow \cos x - 2\sin x$

$$\frac{(2x+1)(\sin x + 2\cos x) - (x^2+x)(\cos x - 2\sin x)}{(\sin x + 2\cos x)^2}$$

$$\frac{2x\sin x + \sin x + 4x\cos x + 2\cos x - x^2\cos x + 2x^2\sin x - x\cos x + 2x\sin x}{(\sin x + 2\cos x)^2}$$

$$\frac{\sin x + 3x\cos x + 2\cos x - x^2\cos x + 2x^2\sin x}{(\sin x + 2\cos x)^2}$$

السؤال الثاني

(i) $y = \sin 3x^2$

$$\frac{dy}{dx} = 6x\cos 3x^2$$

(ii) $y = e^{x^2}$

$$\frac{dy}{dx} = 2x\,e^{x^2}$$

(iii) $\log(1 + x + x^3)$

$$\frac{dy}{dx} = \frac{1 + 3x^2}{(1+x+x^3)\ln 10}$$

السؤال الثالث

(i) $y = y^2 - 3\sin x = x$

$$2y\frac{dy}{dx} - 3\cos x = 1$$

$$\frac{2y}{2y}\frac{dy}{dx} = \frac{3\cos x + 1}{2y}$$

$$\frac{dy}{dx} = \frac{3\cos x + 1}{2y}$$

(ii) $2x^2 - 3\cos y = 4$

$$4x + 3\sin y\frac{dy}{dx} = 0$$

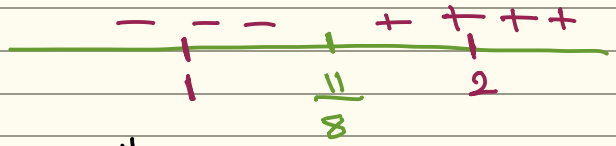
$$\frac{3\sin y}{3\sin y}\frac{dy}{dx} = \frac{-4x}{3\sin y}$$

$$\frac{dy}{dx} = -\frac{4x}{3\sin y}$$

السؤال الرابع

(i) $f(x) = 4x^2 - 11x + 5$ نشتق ونساوي المشتقة بصفر

$f'(x) = 8x - 11 \Rightarrow 8x - 11 = 0 \quad x = \frac{11}{8}$



نعوض في المشتقة بقيمة يمين $\frac{11}{8}$

$f'(2) = 8 \cdot 2 - 11 = 16 - 11 = 5$

ونعوض في المشتقة قيمة يسار $\frac{11}{8}$

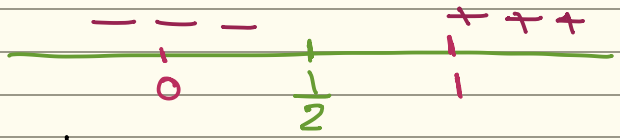
$f'(1) = 8 \cdot 1 - 11 = 8 - 11 = -3$

$x > \frac{11}{8}$, $(\frac{11}{8}, \infty)$ increasing

$x < \frac{11}{8}$, $(-\infty, \frac{11}{8})$ decreasing

(ii) $f(x) = x^2 - x + 8$

$f'(x) = 2x - 1 \Rightarrow 2x - 1 = 0 \Rightarrow x = \frac{1}{2}$



$f'(1) = 2(1) - 1 =$

$2 - 1 = \boxed{1}$

$f'(0) = 2(0) - 1$

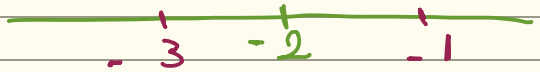
$0 - 1 = \boxed{-1}$

$x > \frac{1}{2}$, $(\frac{1}{2}, \infty)$ increasing

$x < \frac{1}{2}$, $(-\infty, \frac{1}{2})$ decreasing

(iii) $f(x) = 5 - 4x - x^2$

$f'(x) = -4 - 2x \Rightarrow -4 = 2x \Rightarrow x = -2$



$f'(-1) = -4 - 2(-1)$

$= -4 + 2 = -2$

$f'(-3) = -4 - 2(-3)$

$-4 + 6 = +2$

$x > -2$, $(-2, \infty)$ decreasing

$x < -2$, $(-\infty, -2)$ increasing

السؤال الخامس: نساوي المشتقة الثانية بصفر

(i) $f(x) = -3x^2 + 5x + 5$

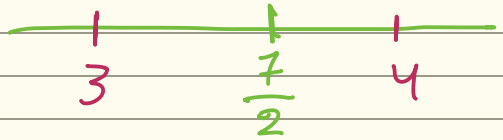
$f'(x) = -6x + 5 \quad f''(x) = -6$

المشتقة الثانية تساوي ثابت 6- اذن الدالة مقعرة للأسفل على مجالها ولا يوجد فيها نقاط تكون فيها مقعرة لأعلى

(ii) $f(x) = x^3 - 7x^2 + 8$

$f'(x) = 3x^2 - 21x$ $\quad f''(x) = 6x - 21$

$6x - 21 = 0$ $\quad x = \frac{21}{6}$ $\quad x = \frac{7}{2}$



$f''(4) = 6 \cdot 4 - 21$

$24 - 21 = 3$

$f''(3) = 6 \cdot 3 - 21$

$18 - 21 = -3$

$\frac{7}{2} < x$, $(\frac{7}{2}, \infty)$ Concave up

$\frac{7}{2} > x$, $(-\infty, \frac{7}{2})$ Concave down

السؤال السادس :-

(i) $f(x) = x^3 - 4x + 5$

$f'(x) = 3x^2 - 4$ $\quad 3x^2 - 4 = 0$ $\quad x^2 = \frac{4}{3}$ $\quad x = \pm \frac{4}{\sqrt{3}}$

$f(\frac{2}{\sqrt{3}}) = (\frac{2}{\sqrt{3}})^3 - 4 \cdot \frac{2}{\sqrt{3}} + 5 = -\frac{16\sqrt{3} - 45}{9}$

$f(\frac{-2}{\sqrt{3}}) = (\frac{-2}{\sqrt{3}})^3 - 4 \cdot \frac{-2}{\sqrt{3}} + 5 = \frac{16\sqrt{3} - 45}{9}$

$(\frac{2}{\sqrt{3}}, \frac{-26\sqrt{3} - 45}{9})$, $(\frac{-2}{\sqrt{3}}, \frac{16\sqrt{3} - 45}{9})$

(ii) $f(x) = 2x^3 - 15x^2 + 24x$

$f'(x) = 6x^2 - 30x + 24 \Rightarrow 6x^2 - 30x + 24 = 0$

$x^2 - 5x + 4 = 0$

$(x - 4)(x - 1) = 0$ $\quad x_1 = 4$, $x_2 = 1$

$f(4) = 2 \cdot (4)^3 - 15(4)^2 + 24(4) = -16$

$f(1) = 2(1)^3 - 15(1)^2 + 24(1) = 11$

$(4, -16)$, $(1, 11)$

(iii) $f(x) = x^2 - 3x + 4$

$f'(x) = 2x - 3 \qquad 2x - 3 = 0 \qquad x = \frac{3}{2}$

$f\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) + 4 = \frac{7}{4}$

$\left(\frac{3}{2}, \frac{7}{4}\right)$

السؤال السابع:

$4x^3 - 12x^2 + 36x$

① نشتق

$f'(x) = 12x^2 - 24x + 36$

② نساوي المشتقة بصفر

$\frac{12}{12}x^2 - \frac{24}{12}x + \frac{36}{12} = 0$

③ نقسم على 12

$x^2 - 2x + 3 = 0$

④ نحل بالقانون العام

$b^2 - 4ac = (-2)^2 - 4 \cdot 1 \cdot 3 = 4 - 12 = -8$

لا توجد قيم لمجهول الدالة

السؤال الثامن:

$f(x) = x^2 + x - 2$

نوجد قيم x

$(x - 1)(x + 2) = 0 \qquad x_1 = 1 \qquad x_2 = 2$

نشتق الدالة

$f'(x) = 2x + 1$

نساوي المشتقة بصفر

$2x + 1 = 0 \qquad x = -\frac{1}{2}$

$c = -\frac{1}{2} \in 1, -\frac{1}{2}$

السؤال التاسع

أثبت أن الدالة تحقق نظرية رول على الفترة

$f(x) = x^2 - 5x + 6 \quad [2, 3]$

$f(2) = (2)^2 - 5(2) + 6 = 4 - 10 + 6 = 0$

$f(3) = (3)^2 - 5(3) + 6 = 9 - 15 + 6 = 0$

$f'(x) = 2x - 5 \Rightarrow 2x - 5 = 0 \quad \boxed{x = \frac{5}{2}}$

$\frac{5}{2} \in [2, 3] \qquad f'(\frac{5}{2}) = 2(\frac{5}{2}) - 5 = 0$

السؤال العاشر

i $\int (2x^4 - 4x^3 + x^{\frac{1}{2}} + \frac{2}{x} - 4)\, dx$

$= \int 2x^4 - 4x^3 + x^{\frac{1}{2}} + 2\frac{1}{x} - 4\, dx$

$= 2\frac{x^5}{5} - \frac{4x^4}{4} + \frac{3x^{\frac{3}{2}}}{2} + 2\ln x - 4x + C$

$\frac{2}{5}x^5 - x^4 + \frac{2}{3}x^{\frac{3}{2}} + 2\ln x - 4x + C$

(ii) $\int \cos(x^2 - 7)\, 2x\, dx$

$= \sin(x^2 - 7) + C$

(iii) $\int \sin^6 x \cos x\, dx$

$\frac{1}{7}\sin^7 x + C$

السؤال الحادي عشر

(i) $\int_0^2 (x + 4x - 2x^3)\, dx$

$\left[\frac{x^2}{2} + \frac{4x^2}{2} - \frac{2x^4}{4}\right]_0^2 = \left[\frac{x^2}{2} + 2x^2 - \frac{1}{2}x^4\right]_0^2$

$\left[\frac{2^2}{2} + 2\cdot 2^2 - \frac{1}{2}2^4\right] - \left[\frac{0^2}{2} + 2(2)^2 - \frac{1}{2}(2)^4\right]$

$\frac{4}{2} + 2\cdot 4 - \frac{1}{2}\cdot 16 = 2$

(8 و −8 تلغيان)

(ii) $\int_0^{\frac{\pi}{6}} \frac{\sin x}{4}$

$$-\frac{1}{4}\cos x\Big]_0^{\frac{\pi}{6}} = \left[-\frac{1}{4}\cos\frac{\pi}{6}\right] - \left[-\frac{1}{4}\cos 0\right]$$

$$\left[-\frac{1}{4}\times\frac{\sqrt{3}}{2}\right] - \left[-\frac{1}{4}\times 1\right] = \frac{\sqrt{3}}{8} + \frac{1}{4}$$

(iii) $\int_{\frac{\pi}{2}}^{\pi} \cos 2x \, dx$

$$\left[\frac{\sin 2x}{2}\right]_{\frac{\pi}{2}}^{\pi} = \frac{\sin 2\pi}{2} - \frac{\sin 2\frac{\pi}{2}}{2} = 0$$

(iv) $\int_0^3 (5x-3)^3 \, dx$

$$\frac{(5x-3)^4}{5\cdot 4}\Big]_0^3 = \left[\left(\frac{5\cdot 3-3}{20}\right)^4 - \left(\frac{5\cdot 0-3}{20}\right)^4\right]$$

$$\left(\frac{15-3}{20}\right)^4 - \left(\frac{-3}{20}\right)^4 = \left(\frac{12}{20}\right)^4 - \left(\frac{-3}{20}\right)^4$$

$$\frac{20736}{20} - \frac{81}{20} = \frac{4131}{4}$$