

Assignment-1
Calculus 1 (Math-105)
First Semester (2024-2025)

Q.1 Find the natural domain of the following functions

(i) $f(x) = \frac{7}{(x-4)(x-5)}$ (ii) $f(x) = \sqrt{x^2 - 7x + 10}$ (iii) $f(x) = 6$

Q.2 (i) Let $f(x) = 7 - x^3$ and $g(x) = x^2$ (ii) Let $f(x) = \cos x$ and $g(x) = x^2$

Find $(f \circ g)x$ and $(g \circ f)x$

Q.3 Check whether the following functions are even or odd

(i) $f(x) = x^2 - \cos x$ (ii) $f(x) = x - \sin x$
(iii) $f(x) = x^2 - \sin x$ (iv) $f(x) = x \sin x$ (v) $f(x) = x \cos x$

Q.4 Find the inverse function from (i) $f(x) = \frac{1}{7}x - 1$ (ii) $f(x) = \sqrt{2x - 5}$

Q.5 Show that the inverse of $f(x) = x^4$ is $f^{-1}(x) = x^{1/4}$

Q.6 Solve each of the following equation for x:

(i) $\ln x = -1$ (ii) $\ln(1 + x^2) - \ln x = \ln 2$ (iii) $e^{3x-4} = 2$

Q.7 Find

(i) $\lim_{x \rightarrow -2} (x^2 + 2x - 3)$ (ii) $\lim_{x \rightarrow -3} \frac{(x^2+6x+9)}{(x^2+x-6)}$ (iii) $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1}$

Q.8 Find

(i) $\lim_{x \rightarrow \infty} \frac{x^2+2x-1}{2x^3+3}$ (ii) $\lim_{x \rightarrow \infty} \frac{6-x^3}{7x^3+3}$ (iii) $\lim_{x \rightarrow \infty} (1 + 2x - 3x^5)$

Q.9 For what values of x is there a discontinuity in the graph of

(i) $y = \frac{x^2-16}{x^2+2x-3}$? (ii) $y = \frac{x+1}{x^2-5x-14}$?

Q.10 Find the horizontal and vertical asymptotes of

(i) $y = \frac{x^2+1}{2x^2-3x-2}$ (ii) $y = \frac{2x^2+x-1}{x^2+x-2}$

السؤال الأول:

$$1) f(x) = \frac{7}{(x-4)(x-5)}$$

$$x-4=0 \quad x=4$$

$$D = \mathbb{R} - \{4, 5\}$$

$$x-5=0 \quad x=5$$

$$(-\infty, 4) \cup (4, 5) \cup (5, \infty)$$

$$2) \sqrt{x^2 - 7x + 10}$$

$$x^2 - 7x + 10 \geq 0$$

$$x-5=0 \quad x=5$$

$$D = (-\infty, 2] \cup [5, \infty)$$

$$x-2=0 \quad x=2$$

$$3) f(x) = 6$$

$$D = \mathbb{R} \quad (-\infty, \infty)$$

$$1) f(x) = 7 - x^3 \quad g(x) = x^2$$

السؤال الثاني:

$$f \circ g = 7 - (x^2)^3 = 7 - x^6$$

$$g \circ f = (7 - x^3)^2 = 49 - 14x^3 + x^6$$

$$2) f(x) = \cos x$$

$$g(x) = x^2$$

$$f \circ g = \cos x^2$$

$$g \circ f = \cos^2 x$$

$$1) f(x) = x^2 - \cos x \quad \text{even}$$

السؤال الثالث:

$$2) f(x) = x - \sin x \quad \text{odd}$$

$$3) f(x) = x^2 - \sin x \quad \text{neither odd nor even}$$

$$4) f(x) = x \sin x \quad \text{even}$$

$$5) f(x) = x \cos x \quad \text{odd}$$

$$\boxed{1} \quad f(x) = \frac{1}{7}x - 1$$

$$f(y) = \left(\frac{1}{7}\right)x - (1)7$$

$$7y = x - 7$$

$$7y + 7 = x$$

$$f^{-1}(x) = 7x + 7$$

$$\boxed{2} \quad f(x) = \sqrt{2x - 5}$$

$$y = \sqrt{2x - 5}$$

$$y^2 = 2x - 5$$

$$y + 5 = 2x$$

$$\frac{y+5}{2} = x \quad f^{-1}(x) = \frac{x+5}{2}$$

السؤال الرابع ..

$$\text{show } f(x) = x^4 \quad ; \quad f^{-1}(x) = x^{\frac{1}{4}}$$

السؤال الخامس

$$y = x^4 \Rightarrow \sqrt[4]{y} = x$$

$$f^{-1}(x) = \sqrt[4]{x} = x^{\frac{1}{4}}$$

$$\boxed{1} \quad \ln x = -1$$

$$e^{\ln x} = e^{-1} \Rightarrow \boxed{x = e^{-1}}$$

السؤال السادس

$$\boxed{2} \quad \ln(1+x^2) - \ln x = \ln 2$$

$$\ln \frac{1+x^2}{x} = \ln 2$$

$$\frac{1+x^2}{x} = 2$$

$$1+x^2 = 2x$$

$$1+x^2-2x=0 \quad = \quad x^2-2x+1=0$$

$$(x-1)^2 = 0 \quad \boxed{x=1}$$

$$\boxed{3} \quad e^{3x-4} = 2$$

$$\ln e^{3x-4} = \ln 2$$

$$3x-4 = \ln 2$$

$$3x = \ln 2 + 4 \rightarrow \boxed{x = \frac{\ln 2 + 4}{3}}$$

السؤال السابع:

$$1) \lim_{x \rightarrow -2} (x^2 + 2x - 3)$$

$$((-2)^2 + 2(-2) - 3) = 4 - 4 - 3 = -3$$

$$2) \lim_{x \rightarrow -3} \frac{x^2 + 6x + 9}{x^2 + x - 6} = \frac{(-3)^2 + 6(-3) + 9}{(-3)^2 + (-3) - 6} = \frac{18 - 18}{9 - 9}$$

$$\frac{0}{0} \lim_{x \rightarrow -3} \frac{(x+3)(x+3)}{(x+3)(x-2)}$$

$$\lim_{x \rightarrow -3} \frac{x+3}{x-2} = \frac{-3+3}{-3-2} = \frac{0}{-5} = 0$$

$$3) \lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1} = \frac{1-1}{1-1} = \frac{0}{0}$$

$$\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} = \frac{x-1 \cdot \sqrt{x}+1}{x-1}$$

$$= \lim_{x \rightarrow 1} \sqrt{x} + 1 = 1 + 1 = 2$$

السؤال الثامن:

$$1) \lim_{x \rightarrow \infty} \frac{x^2 + 2x - 1}{2x^3 + 3} = \frac{\frac{x^2}{x^3} + \frac{2x}{x^3} - \frac{1}{x^3}}{2\frac{x^3}{x^3} + \frac{3}{x^3}} = \boxed{0}$$

$$2) \lim_{x \rightarrow \infty} \frac{6 - x^3}{7x^3 + 3} = \frac{\frac{6}{x^3} - \frac{x^3}{x^3}}{7\frac{x^3}{x^3} + \frac{3}{x^3}} = \boxed{\frac{-1}{7}}$$

$$3) \lim_{x \rightarrow \infty} (1 + 2x - 3x^5) = 1 + 2\infty - 3\infty = \boxed{-\infty}$$

واجاد القيم التي تجعل المالك منفطحة في الرسم

السؤال الرابع :-

$$1) y = \frac{x^2 - 26}{x^2 + 2x - 3}$$

$$2) y = \frac{x + 1}{x^2 - 5x - 14}$$

$$x^2 + 2x - 3 = 0$$

$$x^2 - 5x - 14 = 0$$

$$(x + 3)(x - 1) = 0$$

$$(x - 7)(x + 2) = 0$$

$$x = -3, x = 1$$

$$x = 7, x = -2$$

السؤال الخامس :-

$$1) y = \frac{x^2 + 1}{2x^2 - 3x - 2}$$

$$\lim_{x \rightarrow \infty} \frac{x^2 + 1}{2x^2 - 3x - 2} = \frac{1}{2}, \quad \lim_{x \rightarrow -\infty} \frac{x^2 + 1}{2x^2 - 3x - 2} = \frac{1}{2}$$

$$2) y = \frac{2x^2 + x - 1}{x^2 + x - 2}$$

$$\lim_{x \rightarrow \infty} \frac{2x^2 + x - 1}{x^2 + x - 2} = 2, \quad \lim_{x \rightarrow -\infty} \frac{2x^2 + x - 1}{x^2 + x - 2} = 2$$