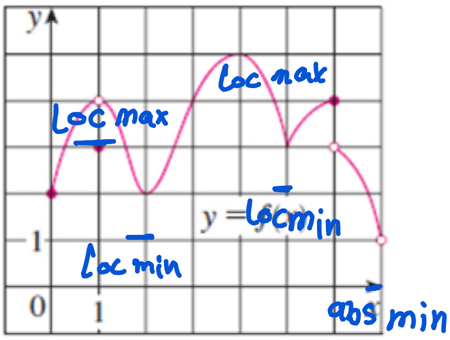
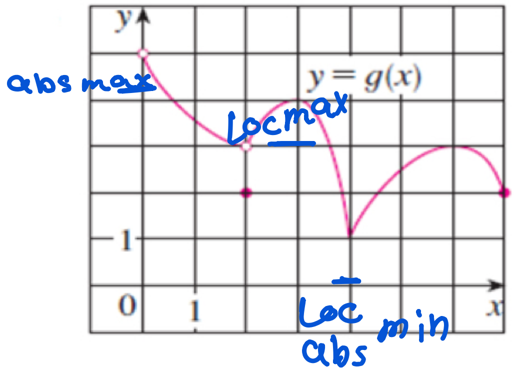
Homework Chapter 4

Q 1. Use the graph to state the absolute and local maximum and minimum values of the function.



(a)



(b)

Q 2. Find the critical number of the function f.

(a). $f(x) = x^3 + x^2 + x$ **(b).** $f(x) = \dfrac{x-1}{x^2 - x + 1}$

(c). $f(x) = \sqrt{1 - x^2}$ **(d).** $f(x) = x^4(x-1)^3$

Q 3. Find the absolute maximum and absolute minimum values of f on the given closed interval.

(a). $f(x) = x^3 - 6x^2 + 9x + 2, \quad -1 \le x \le 4$ **(b).** $f(x) = x^4 - 2x^2 + 3, \quad -2 \le x \le 3$

(c). $f(t) = t.\sqrt{4 - t^2}, \quad -1 \le t \le 2$ **(d).** $f(x) = e^{-x} - e^{-2x}, \quad 0 \le x \le 1$

Q 4. Show that the function $f(x) = x^2 - 8x + 15$ satisfies the hypothesis of the Rolle's theorem over the interval $[3, 5]$, and find all values of c in the interval $(3, 5)$, in which $f'(c) = 0$.

Q 5. Show that the function $f(x) = \dfrac{x}{2} - \sqrt{x}$ satisfies the hypothesis of the Rolle's theorem over the interval $[0, 4]$, and find all values of c in the interval $(0, 4)$, in which $f'(c) = 0$.

Q 7. Show that the function $f(x) = x^3 + x - 4$ satisfies the hypothesis of the Mean-Value theorem over the interval $[-1, 2]$, and find all values of c in the interval $(-1, 2)$ that satisfy the conclusion of the theorem.

Q 8. Show that the function $f(x) = x - \dfrac{1}{x}$ satisfies the hypothesis of the Mean-Value theorem over the interval $[3, 4]$, and find all values of c in the interval $(3, 4)$ that satisfy the conclusion of the theorem.

Q 9. For the given function.

(i). $f(x) = x^2 - 5x + 6$ **(ii).** $f(x) = x^4 - 2x^2 + 3$ **(iii).** $f(x) = \sqrt[3]{x+2}$

Find,

(a). The interval on which f is increasing.
(b). The interval on which f is decreasing.
(c). The inflection point.

(d). Find the local maximum and local minimum values of f.

(e). The open interval on which f is concave upward.

(f). The open interval on which f is concave downward.

Q 10. Sketch the graph of $f(x) = \dfrac{(x+1)^2}{1+x^2}$.

السؤال الثاني: ١. نشتق

٢. نساوي المشتقة بصفر

٣. نوجد قيم x

$\boxed{a}$ $f(x) = x^3 + x^2 + x$

$f'(x) = 3x^2 + 2x + 1$ $\quad 3x^2 + 2x + 1 = 0$

$a = 3 \quad b = 2 \quad c = 1$

$\sqrt{(2)^2 - 4 \cdot 3 \cdot 1} = \sqrt{4 - 12} = \sqrt{-8}$

الداله ليس لها نقاط حرجه

$\boxed{b}$ $f(x) \dfrac{x - 1}{x^2 - x + 1}$

$x - 1 \longrightarrow 1$

$x^2 - x + 1 \longrightarrow 2x - 1$

$f'(x) \dfrac{(x^2 - x + 1) - (x-1)(2x - 1)}{(x^2 - x + 1)^2} = \dfrac{x^2 \cancel{-x} + \cancel{1} - 2x^2 \cancel{+x} + 2x \cancel{-1}}{(x^2 - x + 1)^2}$

$\dfrac{-x^2 + 2x}{(x^2 - x + 1)^2} = 0$ نساوي البسط بصفر

$-x^2 + 2x = 0$ $\quad x_1 = 0 \quad x - 2 = 0 \quad x_2 = 2$

$(0, f(0)),\ (2, f(2)),\ (0, -1),\ (2, \frac{1}{3})$

$\boxed{c}$ $f(x) = \sqrt{1 - x^2}$

$f'(x) = \dfrac{-x}{\sqrt{1 - x^2}} = 0$

$x^2 - 1 = 0$

$x_1 = 0 \quad x_2 = -1 \quad x_2 = 1$

$(0, 1),\ (-1, 0),\ (1, 0)$

$\boxed{d}$ $f(x) = x^4 (x - 1)^3$

$x^4 \longrightarrow 4x^3$

$(x-1)^3 \longrightarrow 3(x-1)^2$

$f'(x) = 4x^3 (x-1)^3 + 3x^4 (x-1)^2$

$4x^3 (x-1)^3 + 3x^4 (x-1)^2 = 0$

$x_1 = 0 \quad x_2 = 1 \quad x_3 = \frac{4}{7}$

$(0, 0),\ (1, 0),\ (\frac{4}{7}, -\frac{6912}{823543})$

السؤال الثالث :.

$\boxed{a}$ $f(x) = x^3 - 6x^2 + 9x + 2 \qquad -1 \leq x \leq 4$

$f'(x) = 3x^2 - 12x + 9$ نساوي المشتقة بصفر

$\frac{3x^2}{3} - \frac{12}{3}x + \frac{9}{3} = 0 \qquad x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$x_1 = 1 \qquad x_2 = 3$

$f(-1) = (-1)^3 - 6(-1)^2 + 9(-1) + 2 = -14$

$f(4) = (4)^3 - 6(4)^2 + 9(4) + 2 = 6$

$f(1) = (1)^3 - 6(1)^2 + 9(1) + 2 = 6$

$f(3) = (3)^3 - 6(3)^2 + 9(3) + 2 = 2$

$f(1)$ = Local maximum

$f(3)$ = Local minimum

$f(4)$ = absolute and Local maximum

$f(-1)$ = absolute minimum

$\boxed{b}$ $f(x) = x^4 - 2x^2 + 3 \qquad -2 \leq x \leq 3$

$f'(x) = 4x^3 - 4x$

$4x = 0 \qquad x^2 - 1 = 0 \qquad x_1 = 0 \quad x_2 = -1 \quad x_2 = 1$

$f(0) = (0)^4 - 2(0)^2 + 3 = 3$

$f(-1) = (-1)^4 - 2(-1)^2 + 3 = 6$

$f(1) = (1)^4 - 2(1)^2 + 3 = 2$

$f(-2) = (-2)^4 - 2(-2)^2 + 3 = 11$

$f(3) = (3)^4 - 2(3)^2 + 3 = 66$

$f(0)$ = local maximum

$f(-1)$ = local minimum

$f(1)$ = local minimum

$f(3)$ = absolt maximum

$\boxed{C}$ $f(t) = t\sqrt{4-t^2}$ $\quad -1 \leq t \leq 2$

$$f'(x) = \frac{-t^2}{\sqrt{4-t^2}} + \sqrt{4-t^2}$$

$t \longrightarrow 1$

$\sqrt{4-t^2} \longrightarrow \frac{-t}{\sqrt{4-t^2}}$

$$= \frac{4-2t^2}{\sqrt{4-t^2}} = 0$$

نساوي المشتقة بصفر

$4-2t^2=0$

عندما البسط يساوي صفر

$2(2-t^2)=0 \quad t^2=2 \Rightarrow t = \pm\sqrt{2}$

$t^2-4=0 \quad t = \pm 2$

عندما المقام يساوي صفر

$f(\sqrt{2}) = \sqrt{2}\sqrt{4-(\sqrt{2})^2} = 2$

$f(-\sqrt{2}) = -\sqrt{2}\sqrt{4-(-\sqrt{2})^2} = -2$

$f(2) = 2\sqrt{4-(2)^2} = 0$

$f(-2) = -2\sqrt{4-(-2)^2} = 0$

$f(-1) = -1\sqrt{4-(-1)^2} = -\sqrt{3} \approx -1,732$

$f(\sqrt{2})$ = local maximum

$f(-1)$ = local minimum

$f(-\sqrt{2})$ = local minimum

$f(2)$, $f(-2)$ = neither a local and absolt maximand minm

$\boxed{D}$ $f(x) = e^{-x} - e^{-2x}$ $\quad 0 \leq x \leq 1$

$f'(x) = -e^{-x} + 2e^{-2x}$

$-e^{-x} + 2e^{-2x} = 0 \quad = -\ln\frac{1}{2}$

$f(-\ln\frac{1}{2}) = -e^{-\ln\frac{1}{2}} + 2e^{-2\ln\frac{1}{2}} = \frac{1}{4}$

$(-\ln\frac{1}{2}, \frac{1}{4})$

$f(0) = e^{-0} - e^{-20} = 0$

$f(1) = -e^{-1} - e^{2(1)} = -0,0972$

$f(0) = 0$ local minimum

$f(1) = -0,0972$ local minimum

$f(\ln\frac{1}{2}) = \frac{1}{4}$ Local maximum

$\boxed{4}$ $f(x) = x^2 - 8x + 15 \quad [3,5]$ السؤال الرابع

$f(3) = (3)^2 - 8(3) + 15 = 9 - 24 + 15 = 0$

$f(5) = (5)^2 - 8(5) + 15 = 25 - 40 + 15 = 0$

$f(3) = f(5)$

$f'(x) = 2x - 8 \qquad 2x - 8 = 0$

$2x = 8 \qquad \boxed{x = 4}$

$f'(4) = 2 \cdot 4 - 8 = 8 - 8 = 0$

$4 \in [3,5]$ الداله تحقق نظرية رول

$\boxed{5}$ $f(x) = \frac{x}{2} - \sqrt{x} \quad [0,4]$ السؤال الخامس

$f(0) = \frac{1}{2}(0) - \sqrt{0} = 0 - 0 = 0$

$f(4) = \frac{1}{2} \cdot 4 - \sqrt{4} = 2 - 2 = 0$

$f(0) = f(4)$

$f'(x) = \frac{1}{2} - \frac{1}{2\sqrt{x}}$

$\frac{1}{2} - \frac{1}{2\sqrt{x}} = 0 \qquad \frac{1}{2} = \frac{1}{2\sqrt{x}}$

$2\sqrt{x} = 2 = 4x = 4 \qquad x = \frac{4}{4} \qquad \boxed{x = 1}$

$f'(1) = \frac{1}{2} - \frac{1}{2\sqrt{1}} = \frac{1}{2} - \frac{1}{2} = 0$

$1 \in [0,4]$ الداله تحقق نظرية رول

السؤال السابع

7 $f(x) = x^3 + x - 4 \quad [-1, 2]$

$f'(x) = 3x^2 + 1$

$f(2) = (2)^3 + (2) - 4 = 8 + 2 - 4 = 6$

$f(-1) = (-1)^3 + (-1) - 4 = -1 - 1 - 4 = -6$

نعوض في القانون

$\frac{f(b) - f(a)}{b - a} = \frac{6 - -6}{2 - -1} = \frac{12}{3} = \boxed{4}$

نساوي المشتقة بـ 4

$3x^2 + 1 = 4 \qquad 3x^2 = 4 - 1$

$3x^2 = 3 \qquad x^2 = 1 \qquad \boxed{x = \pm 1}$

$+1 \in [-1, 2] \quad , \quad -1 \in [-1, 2]$

الدالة تحقق نظرية القيمة المتوسطة

السؤال الثامن

8 $f(x) = x - \frac{1}{x} \quad [3, 4]$

$f'(x) = 1 + \frac{1}{x^2}$

$f(4) = 4 + \frac{1}{4} = \frac{16 - 1}{4} = \frac{15}{4}$

$f(3) = 3 + \frac{1}{3} = \frac{9 - 1}{3} = \frac{8}{3}$

نعوض في القانون

$\frac{f(b) - f(a)}{b - a} = \frac{\frac{15}{4} - \frac{8}{3}}{4 - 3}$

$= \frac{15}{4} - \frac{8}{3} = \frac{45 - 32}{12} = \frac{13}{12} = \boxed{1,08\bar{3}}$

$1,083 \in [3, 4]$

الدالة تحقق نظرية القيمة المتوسطة

9

السؤال التاسع

(i) $f(x) = x^2 - 5x + 6$

$f'(x) = 2x - 5$

$f''(x) = 2$

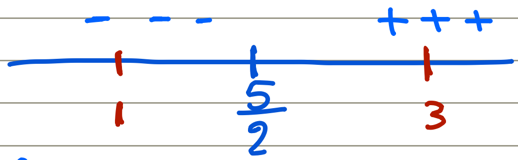
المشتقة الثانية تساوي عدد موجب.

- الدالة مقعرة للأعلى

- لا توجد نقطة انقلاب

$2x - 5 = 0$

$x = \frac{5}{2}$



$f'(1) = 2(1) - 5 = -3$

$f'(3) = 2\cdot3 - 5 = 1$

$(-\infty, \frac{5}{2})$ متناقصة

$(\frac{5}{2}, \infty+)$ متزايدة

$f(\frac{5}{2}) = (\frac{5}{2})^2 - 5(\frac{5}{2}) + 6 = -2$

$f(\frac{5}{2}) = -2$ $(\frac{5}{2}, -2)$ Local mimum

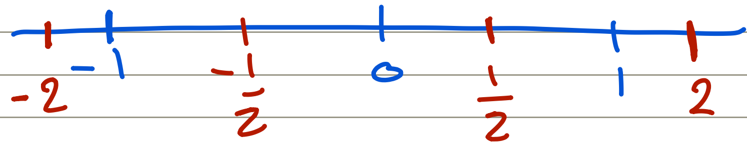
(ii) $f(x) = x^4 - 2x^2 + 3$

$f'(x) = 4x^3 - 4x$

$4x^3 - 4x = 0$

$4x(x^2 - 1)$

$x_1 = 0$ $x_2 = 1$ $x_3 = -1$



$f'(-2) = 4(-2)^3 - 4(-2) = -32 + 8 = -24$

$f\left(-\frac{1}{2}\right) = 4\left(-\frac{1}{2}\right)^3 - 4\left(-\frac{1}{2}\right) = -\frac{1}{2} + 2 = \frac{3}{2}$

$f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 4\left(-\frac{1}{2}\right) = \frac{1}{2} - 2 = \frac{-3}{2}$

$f(2) = 4(2)^3 - 4(2) = 32 - 8 = 24$

$(-\infty, -1) \cup (0, 1)$ متناقصه

$(-1, 0) \cup (1, \infty+)$ متزايده

$f(-1) = (-1)^4 - 2(-1)^2 + 3 = 1 - 2 + 3 = 2 \rightarrow$ Local mimum

$f(0) = (0)^4 - 2(0) + 3 = 3 \rightarrow$ Local maximum

$f(1) = (1)^4 - 2(1) + 3 = 1 - 2 + 3 = 1 + 1 = 2 \rightarrow$ Local mimum

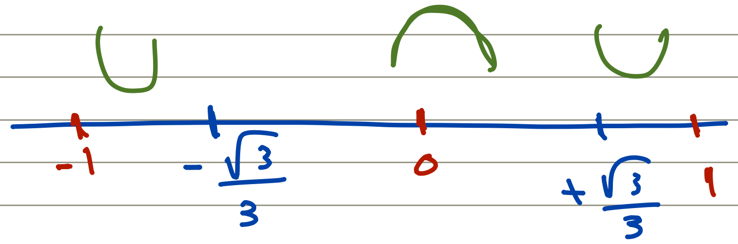
$f''(x) = 12x^2 - 4$

$12x^2 - 4 = 0 \qquad 4(3x^2 - 1) = 0$

$3x^2 - 1 = 0 \qquad x^2 = \frac{1}{3} \qquad \sqrt{x^2} = \pm\frac{1}{\sqrt{3}}$

$\boxed{\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \quad \frac{\sqrt{3}}{3}}$

$x_1 = \frac{\sqrt{3}}{3} \qquad x_2 = -\frac{\sqrt{3}}{3}$



$f''(-1) = 12(-1)^2 - 4 = 12 - 4 = 8$

$f''(0) = 12(0)^2 - 4 = 0 - 4 = -4$

$f''(1) = 12(1)^2 - 4 = 12 - 4 = 8$

$\left(-\infty, -\frac{\sqrt{3}}{3}\right) \cup \left(\frac{\sqrt{3}}{3}, \infty\right)$ مقعرة للأعلى

$\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$ مقعرة للأسفل

$$f\left(-\frac{\sqrt{3}}{3}\right) = \left(-\frac{\sqrt{3}}{3}\right)^4 - 2\left(-\frac{\sqrt{3}}{3}\right)^2 + 3 = \frac{22}{9}$$

$$f\left(\frac{\sqrt{3}}{3}\right) = \left(\frac{\sqrt{3}}{3}\right)^4 - 2\left(\frac{\sqrt{3}}{3}\right)^2 + 3 = \frac{22}{9}$$

$\left(\frac{\sqrt{3}}{3}, \frac{22}{9}\right), \left(\frac{\sqrt{3}}{3}, \frac{22}{9}\right)$ نقطتي انقلاب

(ii) $f(x) = \sqrt[3]{x+2}$ $\quad (x+2)^{\frac{1}{3}}$

$$f'(x) = \frac{1}{3}(x+2)^{-\frac{2}{3}}$$

$f'(x) = 0 \qquad \frac{1}{3}(x+2)^{-\frac{2}{3}} = 0$

$$\boxed{x = -2}$$

$+ + + \quad |_{-2} \quad + + +$

$$f'(-1) = \frac{1}{3}(-1+2)^{-\frac{2}{3}} = \frac{1}{3}$$

$$f(-3) = \frac{1}{3}(-3+2)^{-\frac{2}{3}} = \frac{1}{3}$$

$(-\infty, -2) \cup (-2, \infty+)$ متزايدة

الدالة متزايدة دائماً

$f(-2) = 0$ $\quad (-2, 0)$ النقطة الحرجة

لا توجد قيم قصوى أو دنيا للدالة

الدالة غير مقعرة لأعلى أو أسفل

لا توجد نقطة انقلاب

$\boxed{10}$ $f(x) = \dfrac{(x+1)^2}{1+x^2}$ السؤال العاشر

$$f'(x) = \frac{2-2x^2}{(1+x^2)^2}$$

$$\frac{2-2x^2}{(1+x^2)^2} = 0 \qquad x_1 = -1 \qquad x_2 = 1$$

$$\begin{array}{ccccc} - - - & & + + + & & - - - \\ -2 & -1 & 0 & +1 & 2 \end{array}$$

$$f'(-2) = \frac{2-2(-2)^2}{(1+(2)^2)^2} = \frac{2-8}{(1+4)^2} = \frac{-6}{25}$$

$$f'(0) = \frac{2-(0)^2}{(1+(0)^2)^2} = \frac{2}{1} = 2$$

$$f(2) = \frac{2-2(2)^2}{(1+(2)^2)^2} = \frac{2-8}{(1+4)^2} = \frac{-6}{25}$$

$(-\infty,-1) \cup (1,\infty+)$ متناقصه

$(-1,1)$ متزايده

$$f(1) = \frac{(1+1)^2}{1+1^2} = \frac{2^2}{2} = \frac{4}{2} = 2 \qquad f(1) = 2$$

$$f(-1) = \frac{(1+-1)^2}{1+(-1)^2} = \frac{(0)^2}{2} = 0 \qquad f(-1) = 0$$

$(1,2)$, $(-1,0)$ النقاط الحرجه

$f(1)$ Local maximum

$f(-1)$ Local minimum

$$f''(x) = \frac{4x(x^2-3)}{(x^2+1)}$$

لا يوجد تقعر للأعلى أو للأسفل للدالة

$$\frac{4x(x^2-3)}{(x^2+1)^3} = 0$$

$x_1 = 0 \qquad x_2 = -\sqrt{3} \qquad x_3 = \sqrt{3}$

$f(0) = 1$

$f(-\sqrt{3}) = \frac{2-\sqrt{3}}{2}$

$f(\sqrt{3}) = \frac{2+\sqrt{3}}{2}$

نقاط الانقلاب

$(0,1)\,,\left(-\sqrt{3}, \frac{2-\sqrt{3}}{2}\right),\left(\sqrt{3}, \frac{2+\sqrt{3}}{2}\right)$