Homework Chapter 5

Q 1. Evaluate the following integral.

(a). $\displaystyle\int \frac{x^3 - 2\sqrt{x}}{x} dx$ **(b).** $\displaystyle\int (1-t).(2+t^2)\,dt$ **(c).** $\displaystyle\int \left(x^2 + 1 + \frac{1}{x^2+1}\right.$

(d). $\displaystyle\int (\csc^2 t - 2e^t)\,dt$ **(e).** $\displaystyle\int \frac{\sin^2 2x}{\sin x}.dx$ **(f).** $\displaystyle\int \left(\sqrt[3]{x^3} + \sqrt[3]{x^2}\right) dx$

(g). $\displaystyle\int x.\left(\sqrt[3]{x} + \sqrt[4]{x}\right) dx$ **(h).** $\displaystyle\int \frac{y + 5y^7}{y^3}.dy$

Q 2. Evaluate the following integral.

(a). $\displaystyle\int_1^4 \sqrt{x}dx$ **(b).** $\displaystyle\int_0^2 (x^3 - 3x + 3)\,dx$ **(c).** $\displaystyle\int_\pi^{2\pi} (x - 2\sin x)\,dx$

(d). $\displaystyle\int_{-1}^3 (3-2x)\,dx$ **(e).** $\displaystyle\int_1^2 (1+2y)^2\,dy$ **(f).** $\displaystyle\int_0^1 \frac{4}{1+t^2}dt$

(g). $\displaystyle\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{6}{\sqrt{1-t^2}}dt$ **(h).** $\displaystyle\int_0^\pi f(x)dx$, where $f(x) = \begin{cases} \sin x, & \text{if } 0 \le x \le \frac{\pi}{2} \\ \cos x, & \text{if } \frac{\pi}{2} \le x \le \pi \end{cases}$

Q 3. Evaluate the following integral by using substitution method.

(a). $\displaystyle\int \frac{x}{\sqrt{1-4x^2}}dx$ **(b).** $\displaystyle\int x^2.(x^3+5)^8\,dx$ **(c).** $\displaystyle\int \cos^3 x.\sin x dx$

(d). $\displaystyle\int [\csc(\sin x)]^2.\cos x dx$ **(e).** $\displaystyle\int \sec^2(4x+1)dx$ **(f).** $\displaystyle\int (1 + \sin t$

(g). $\displaystyle\int e^{\sin x}.\cos x dx$ **(h).** $\displaystyle\int e^{-5x}dx$ **(i).** $\displaystyle\int \frac{x}{(x^2+1)^2}dx$

(j). $\displaystyle\int \sec 2\theta.\tan 2\theta dx$ **(k).** $\displaystyle\int e^{\tan x}.\sec^2 x dx$ **(l).** $\displaystyle\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}dx$

Q 4. Evaluate the following integral by using integration by part.

(a). $\displaystyle\int x.\cos x dx$ **(b).** $\displaystyle\int x.\ln(x)dx$ **(c).** $\displaystyle\int t.e^t dx$

(d). $\displaystyle\int e^x.\cos x dx$ **(e).** $\displaystyle\int_0^1 \cot^{-1} x dx$ **(f).** $\displaystyle\int \ln x.e^x dx$

Q (1) السؤال الأول:

a $\int \frac{x^3 - 2\sqrt{x}}{x}\,dx = \int \frac{x^3}{x} - \frac{2\sqrt{x}}{x}\,dx$

$= \int x^2 - 2\sqrt{x}\cdot x^{-1}\,dx = \frac{x^3}{3} - 4\sqrt{x} + C$

b $\int (1-t)(2+t^2)\,dt = \int 2 + t^2 - 2t - t^3\,dt$

$= 2t + \frac{1}{3}t^3 - t^2 - \frac{1}{4}t^4 + C$

c $\int \left(x^2 + 1 + \frac{1}{x^2+1}\right) dx$

$= \frac{x^3}{3} + x + \tan^{-1}x + C$

d $\int (\csc^2 t - 2e^t)\,dt$

$-\cot(t) - 2e^t + C$

e $\int \frac{\sin 2x}{\sin x}\,dx$

$-\frac{4\cos x^3}{3} + C$

f $\int \sqrt[3]{x^3} + \sqrt[3]{x^2}\,dx = \int x^{\frac{3}{3}} + x^{\frac{2}{3}}\,dx$

$= \frac{x^2}{2} + \frac{3\sqrt[3]{x^2}}{5} + C$

g $\int x\left(\sqrt[3]{x} + \sqrt[4]{x}\right) dx = \int x\cdot x^{\frac{1}{3}} + x\cdot x^{\frac{1}{4}}\,dx$

$= \int x^{\frac{4}{3}} + x^{\frac{5}{4}}\,dx$

$$= \frac{3}{7} x^{\frac{7}{3}} + \frac{4}{9} x^{\frac{9}{4}} + C = \frac{3}{7} \sqrt[3]{x^7} + \frac{4}{9} \sqrt[4]{x^9} + C$$

$\boxed{h}$ $$\int \frac{y + 5y^7}{y^3} \, dy = \int \frac{y}{y^3} + \frac{5y^7}{y^3} \, dy$$

$$= \int y^{-2} + 5y^4 \, dy = \frac{-1}{y} + y^5 + C$$

Q(2) السؤال الثاني

$\boxed{a}$ $$\int_1^4 \sqrt{x} \, dx = \int_1^4 x^{\frac{1}{2}} \, dx = \frac{2}{3} \sqrt{x^3} \Big]_1^4$$

$$= \left(\frac{2(\sqrt{4})^3}{3} - \frac{2(\sqrt{1})}{3} \right) = \frac{14}{3}$$

$\boxed{b}$ $$\int_0^2 x^3 - 3x + 3 \, dx$$

$$\frac{x^4}{4} - \frac{3x^2}{2} + 3x \Big]_0^2 = \left(\frac{2^4}{4} - \frac{3 \cdot (2)^2}{2} + 3 \cdot 2 \right) - \left(\frac{0^4}{4} - \cancel{\frac{3 \cdot 0^2}{2}} + 3 \cdot 0 \right)$$

$$= \frac{16}{4} - \frac{12}{2} + 6 = 4$$

$\boxed{c}$ $$\int_\pi^{2\pi} (x - 2\sin x) \, dx = \frac{x^2}{2} + 2\cos x \Big]_\pi^{2\pi}$$

$$= \left[\frac{(2\pi)^2}{2} + 2\cos 2\pi \right] - \left[\frac{(\pi)^2}{2} + 2\cos \pi \right]$$

$$3 \frac{\pi^2}{2} + 4$$

$\boxed{d}$ $\int_{-1}^{3} (3-2x)\,dx = 3x - x^2 \Big]_{-1}^{3}$

$\left[3\cdot 3 - (3)^2\right] - \left[3(-1) - (-1)^2\right]$

$= \cancel{9} \cancel{-9} - (-3-1) = 4$

$\boxed{e}$ $\int_{1}^{2} (1+2y)^2\,dy$

نفك المربع التام قبل الحل

$\int_{1}^{2} 1 + 4y + 4y^2\,dy = y + \frac{\cancel{4}^{2}}{\cancel{2}} y^2 + \frac{4}{3} y^3 \Big]_{1}^{2}$

$\left[2 + 2\cdot(2)^2 + \frac{4}{3}(2)^3\right] - \left[1 + 2\cdot(1)^2 + \frac{4}{3}(1)^3\right]$

$= \left[2 + 8 + \frac{32}{3}\right] - \left[1 + 3 + \frac{4}{3}\right] = 10 + \frac{32}{3} - 4 + \frac{4}{3} = \frac{49}{3}$

$\boxed{f}$ $\int_{0}^{1} \frac{4}{1+t^2}\,dt \quad 4\int_{0}^{1} \frac{1}{1+t^2}\,dt$

$= 4\tan^{-1} t \Big]_{0}^{1} = \left[4\tan^{-1} 1\right] - \left[4\tan^{-1} 0\right] = \pi$

$\boxed{g}$ $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{6}{\sqrt{1-t^2}}\,dt = 6\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-t^2}}\,dt$

$= 6\sin^{-1} t \Big]_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} = \left[6\sin^{-1}\frac{\sqrt{3}}{2} - 6\sin^{-1}\frac{1}{2}\right] = \pi$

$\boxed{d}$ $\int_0^{\pi} f(x)\,dx$ $f(x) = \sin x \quad 0 \le x \le \frac{\pi}{2}$

$f(x) \; \cos x \quad \frac{\pi}{2} \le x \le \pi$

نقسم التكامل إلى جزأين بناءً على التعريف

$$\int_0^{\pi} f(x)\,dx = \int_0^{\frac{\pi}{2}} \sin x \,dx + \int_{\frac{\pi}{2}}^{\pi} \cos x\,dx$$

التكامل الأول

$$= \left[-\cos x\right]_0^{\frac{\pi}{2}} = -\cos\left(\frac{\pi}{2}\right) + \cos(0) = 0 + 1 = 1$$

التكامل الثاني:

$$= \left[\sin x\right]_{\frac{\pi}{2}}^{\pi} = \sin(\pi) - \sin\frac{\pi}{2} = 0 - 1 = -1$$

نجمع النواتج:

$$1 + -1 = 0$$

$$\therefore \int_0^{\pi} f(x)\,dx = 0$$

Q (3) السؤال الثالث:

$\boxed{a}$ $$\int \frac{x}{\sqrt{1-4x^2}}\,dx$$

$$= \frac{1}{2}\sin^{-1} 2x + c$$

$\boxed{b}$ $$\int x^2 (x^3+5)^8\,dx$$

$$\frac{1}{3}\,\frac{(x^3+5)^9}{9} + C = \frac{(x^3+5)^9}{27} + c$$

$\boxed{c}$ $\int \cos^3 x \sin x \, dx$

$$-\frac{\cos^4 x}{4} + C$$

$\boxed{d}$ $\int [\csc(\sin x)]^2 \cos x \, dx$

$$= -\cot(\sin x) + C$$

$\boxed{e}$ $\int \sec^2(4x+1) \, dx$

$$= \frac{\tan 4x+1}{4} + C$$

$\boxed{f}$ $\int (1+\sin t)^9 \cos t \, dt$

$$\frac{(1+\sin t)^{10}}{10} + C$$

$\boxed{g}$ $\int e^{\sin x} . \cos x \, dx$

$$e^{\sin x} + C$$

$\boxed{h}$ $\int e^{-5x} \, dx$

$$\frac{e^{-5x}}{5} + C$$

$\boxed{i}$ $\int \frac{x}{(x^2+1)^2}\,dx$

$-\frac{1}{2x^2+2} + c$

$\boxed{j}$ $\int \sec 2\theta \; \tan 2\theta \; dx$

$= \frac{\sin 2\theta\, x}{\cos(2\theta)^2} + c$

$\boxed{k}$ $\int e^{\tan x} \cdot \sec^2 x \, dx$

$e^{\tan x} + c$

$\boxed{l}$ $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx$

$= \frac{\sin^{-1} x^{2}}{2} + c$

Q(4) السؤال الرابع

$\boxed{a}$ $\int x\cos x \, dx$

$x \xrightarrow{\text{اشتقاق}} 1$

$\cos x \xrightarrow{\text{تكامل}} \sin x$

$x\sin x - \int \sin x\, dx$

$x\sin x + \cos x + c$

$\boxed{b}$ $\int x \ln x \, dx$

$\ln x \xrightarrow{\text{اشتقاق}} \frac{1}{x}$

$x \xrightarrow{\text{تكامل}} \frac{1}{2}x^2$

$\frac{1}{2}x^2 \ln x - \int \frac{1}{2} x^{\cancel{2}} \cdot \frac{1}{\cancel{x}} \, dx$

$\frac{1}{2}x^2 \ln x - \frac{1}{2}\int x \, dx$

$\frac{1}{2}x^2 \ln x - \frac{1}{2}\frac{x^2}{2} + C = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$

$\boxed{c}$ $\int t e^t \, dt$

$t \xrightarrow{\text{اشتقاق}} 1$

$e^t \xrightarrow{\text{تكامل}} e^t$

$t e^t - \int e^t \, dt$

$t e^t - e^t + C$

$\boxed{d}$ $\int e^x \cos x \, dx$

$\frac{\cos x \, e^x + \sin x \, e^x}{2} + C$

$\boxed{e}$ $\int_0^1 \cot^{-1} x \, dx$

$= \frac{\pi}{4} + \frac{1}{2}\ln(2)$

$\boxed{f}$ $\int \ln x \, e^x \, dx$

$e^x \ln x - Ei(x) + C$