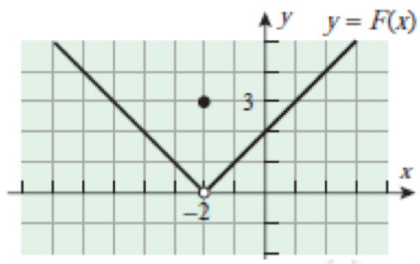
Homework Chapter 2

Q 1. Find the function F graphed in the figure below:

(a) $\lim\limits_{x\longrightarrow -2^-} F(x)$, 0 **(b)** $\lim\limits_{x\longrightarrow -2^+} F(x)$ 0

(a) $\lim\limits_{x\longrightarrow -2} F(x)$, 0 **(d)** $F(-2)$ 3



Q 2. Find the limit, if it exists.

(a) $\lim\limits_{x\longrightarrow 2} x(x-1)(x+1)$, **(b)** $\lim\limits_{x\longrightarrow -1} \sqrt{x^4+3x+6}$, **(c)** $\lim\limits_{x\longrightarrow 2} \dfrac{2x^2+1}{x^2+6x-4}$

(d) $\lim\limits_{x\longrightarrow 2} \dfrac{x^2+x-6}{x-2}$ **(e)** $\lim\limits_{x\longrightarrow 3} \dfrac{x^2-9}{x-3}$ **(f)** $\lim\limits_{x\longrightarrow 1} \dfrac{x-1}{\sqrt{x}-1}$

(g) $\lim\limits_{x\longrightarrow 0} \dfrac{\sqrt{x+4}-2}{x}$ **(h)** $\lim\limits_{x\longrightarrow 2^+} \dfrac{1}{|2-x|}$ **(i)** $\lim\limits_{x\longrightarrow 3^-} \dfrac{1}{|x-3|}$

Q 3. Determine the infinite limits

Q_2 a) $\lim\limits_{x\to 2} x(x-1)(x+1) = 2(2-1)(2+1)$

$= 2(1)(3) = 6$

b) $\lim\limits_{x\to -1} \sqrt{x^4+3x+6} = \sqrt{(-1)^4+3(-1)+6}$

$= \sqrt{1-3+6} = \sqrt{4} = 2$

c) $\lim\limits_{x\to 2} \dfrac{2x^2+1}{x^2+6x-4} = \dfrac{2(2)^2+1}{(2)^2+6\cdot 2-4} = \dfrac{2\cdot 4+1}{\cancel{4}+12-\cancel{4}}$

$= \dfrac{8+1}{12} = \dfrac{9^{\div 3}}{12_{\div 3}} = \dfrac{3}{4}$

d) $\lim\limits_{x\to 2} \dfrac{x^2+x-6}{x-2} = \dfrac{2^2+2-6}{2-2} = \dfrac{6-6}{2-2} = \dfrac{0}{0}$

$\lim\limits_{x\to 2} \dfrac{\cancel{(x-2)}(x+3)}{\cancel{x-2}} = \lim\limits_{x\to 2} x+3 = 2+3 = 5$

e) $\lim\limits_{x\to 3} \dfrac{x^2-9}{x-3} = \dfrac{3^2-9}{3-3} = \dfrac{9-9}{3-3} = \dfrac{0}{0}$

$\lim\limits_{x\to 3} \dfrac{\cancel{(x-3)}(x+3)}{\cancel{x-3}} = \lim\limits_{x\to 3} x+3 = 3+3 = 6$

f) $\lim\limits_{x\to 1} \dfrac{x-1}{\sqrt{x}-1} = \dfrac{1-1}{\sqrt{1}-1} = \dfrac{0}{0}$

$\lim\limits_{x\to 1} \dfrac{x-1}{\sqrt{x}-1} \cdot \dfrac{\sqrt{x}+1}{\sqrt{x}+1} = \dfrac{x-1 \cdot \sqrt{x}+1}{(\sqrt{x})^2-(1)^2}$

$\lim\limits_{x\to 1} \dfrac{\cancel{x-1}\cdot\sqrt{x}+1}{\cancel{x-1}} = \lim\limits_{x\to 1} \sqrt{x}+1 = 1+1 = 2$

g) $\lim\limits_{x\to 0} \dfrac{\sqrt{x+4}-2}{x} = \dfrac{\sqrt{0+4}-2}{0} = \dfrac{2-2}{0-0} = \dfrac{0}{0}$

$\lim\limits_{x\to 0} \dfrac{\sqrt{x+4}-2}{x} \cdot \dfrac{\sqrt{x+4}+2}{\sqrt{x+4}+2}$

$= \lim\limits_{x\to 0} \dfrac{(\sqrt{x+4})^2-(2)^2}{x\sqrt{x+4}+2} = \lim\limits_{x\to 0} \dfrac{\cancel{x}+\cancel{4}-\cancel{4}}{\cancel{x}\sqrt{x+4}+2}$

$= \lim\limits_{x\to 0} \dfrac{1}{\sqrt{x+4}+2} = \dfrac{1}{\sqrt{0+4}+2} = \dfrac{1}{2+2} = \dfrac{1}{4}$

h) $\lim\limits_{x\to 2^+} \dfrac{1}{|2-x|} = \dfrac{1}{|2-2|} = \dfrac{1}{0} = \infty$

i) $\lim\limits_{x\to 3^-} \dfrac{1}{|x-3|} = \dfrac{1}{|3-3|} = \dfrac{1}{0} = \infty$

Q 3. Determine the infinite limits.

(a) $\lim\limits_{x\longrightarrow 2^-} \dfrac{x}{|x^2-4|}$ (b) $\lim\limits_{x\longrightarrow 2^+} \dfrac{x}{|x^2-4|}$ (c) $\lim\limits_{x\longrightarrow 2} \dfrac{x}{|x^2-4|}$

(d) $\lim\limits_{x\longrightarrow -3^-} \dfrac{x+2}{x+3}$ (e) $\lim\limits_{x\longrightarrow -3^+} \dfrac{x+2}{x+3}$ (f) $\lim\limits_{x\longrightarrow 1} \dfrac{2-x}{(x-1)^2}$

Q 3

a) $\lim\limits_{x\to 2^-} \dfrac{x}{|x^2-4|} = \dfrac{2}{|4-4|} = \dfrac{2}{0} = \infty$

b) $\lim\limits_{x\to 2^+} \dfrac{x}{|x^2-4|} = \dfrac{2}{|2^2-4|} = \dfrac{2}{0} = \infty$

c) $\lim\limits_{x\to 2} \dfrac{x}{|x^2-4|} = \dfrac{2}{|4-4|} = \dfrac{2}{0} = \infty$

الداله تقترب من اليسار

d) $\lim\limits_{x\to -3^-} \dfrac{x+2}{x+3} = \dfrac{-3+2}{-3+3} = \dfrac{-1}{0} = \infty$

الداله تقترب من اليمين

e) $\lim\limits_{x\to -3^+} \dfrac{x+2}{x+3} = \dfrac{-3+2}{-3+3} = -\dfrac{1}{0} = -\infty$

f) $\lim\limits_{x\to 1} \dfrac{2-x}{(x-1)^2} = \dfrac{2-1}{(x-1)^2} = \dfrac{1}{0} = \infty$

Q 4. Let $f(x) = \begin{cases} x-1, & \text{if } x \le 3 \\ 3x-7, & \text{if } x > 3 \end{cases}$

Find,

(a) $\lim\limits_{x\longrightarrow 3^-} f(x)$ (b) $\lim\limits_{x\longrightarrow 3^+} f(x)$ (c) $\lim\limits_{x\longrightarrow 3} f(x)$.

Q 4

$f(x) \begin{cases} x-1 & \text{if } x \le 3 \\ 3x-7 & \text{if } x > 3 \end{cases}$

find a) $\lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3^-} x-1 = 3-1 = 2$

b) $\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3^+} 3x-7 = 3.3-7$

$= 9-7 = 2$

c) $\lim\limits_{x\to 3} f(x) = 2$

$\lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3}$ Exact

Q5 . By using Squeeze theorem, show that

(a) $\lim_{x \to 0} x^2 \cos\left(\frac{2}{x}\right) = 0$ **(b)** $\lim_{x \to 0} x^2 e^{\sin\left(\frac{1}{x}\right)} = 0$.

a) $\lim_{x \to 0} x^2 \cos\left(\frac{2}{x}\right) = 0$

$-1 \leq \cos x \leq +1$

$-1 \cdot x^2 \leq x^2 \cos \frac{2}{x} \leq 1 \cdot x^2$ → نضرب كل الحدود x^2

$-x^2 \leq x^2 \cos \frac{2}{x} \leq x^2$

$\lim_{x \to 0} (-x)^2 = (-0)^2 = 0$ $\lim_{x \to 0} x^2 = (0)^2 = 0$

$\therefore \lim_{x \to 0} x^2 \cos \frac{2}{x} = 0$ من نظرية الساندويتش

b) $\lim_{x \to 0} x^2 e^{\sin \frac{1}{x}} = 0$

$e^{-1} \leq e^{\sin \frac{1}{x}} \leq e^{1}$ نضيف لكل الحدود e

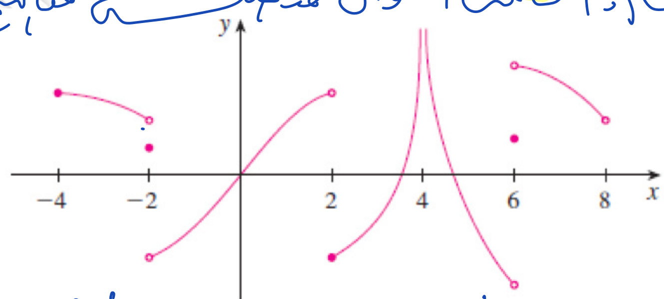
$x^2 e^{-1} \leq x^2 e^{\sin \frac{1}{x}} \leq x^2 e^{1}$ نضرب كل الحدود x^2

$\lim_{x \to 0} x^2 e^{-1} = (0) e^{-1} = 0$ $\lim_{x \to 0} x^2 e^{1} = (0) e^{1} = 0$

$\therefore \lim_{x \to 0} x^2 e^{\sin \frac{1}{x}} = 0$

Q 6. Determine whether f is continuous from the right, or from the left, or neither.

حدد ما إذا كانت الدوال متصلة من اليمين أو اليسار



الدالة غير متصلة من اليمين أو اليسار عند النقاط الموضحة

Q 7. Where are each of the following functions discontinuous?

(a). $f(x) = x^2 + 2x - 4$ **(b).** $f(x) = \sqrt[3]{x-8}$ **(c).** $f(x) = \dfrac{x+2}{x^2-4}$

(d). $f(x) = \dfrac{3x^2-2}{4-x}$ **(e).** $f(x) = \dfrac{x}{2x^2+x}$ **(f).** $f(x) = \begin{cases} \dfrac{x^2-x}{x^2-1}, & \text{if } x \neq 1 \\ 1, & \text{if } x = 1 \end{cases}$

(g). $f(x) = \begin{cases} (2x+3), & \text{if } x \leq 4 \\ 7 + \dfrac{16}{x}, & \text{if } x > 4 \end{cases}$

Q7 أي من الدوال التالية غير متصلة

a) $f(x) = x^2 + 2x - 4$ دالة متصلة

b) $f(x) = \sqrt[3]{x-8}$ دالة متصلة

c) $f(x) = \dfrac{x-4}{x^2-4}$ الدالة منفصلة إلا عند $x = \pm 2$

$x^2 - 4 = (x-2)(x+2)$

is not continuous at $x = \{-2, +2\}$

g) $f(x) \begin{cases} (2x+3) & \text{if } x \leq 4 \\ 7 + \dfrac{16}{x} & \text{if } x > 4 \end{cases}$

$f(4) = 2x + 3 = 2 \cdot 4 + 3 = 11$

$\lim\limits_{x \to 4^-} f(x) = 2x + 3 = 2 \cdot 4 + 3 = 11$

$\lim\limits_{x \to 4^+} f(x) = 7 + \dfrac{16}{4} = 7 + 4 = 11$

is continuous at $x = 4$

الدالة متصلة عند $x = 4$

لأن $\lim\limits_{x \to 4^+} f(x) = \lim\limits_{x \to 4^-} f(x) = f(4)$

d) $\dfrac{3x^2-2}{4-x}$ الدالة متصلة إلا عند $x = 4$

$4 - x = 0 \Rightarrow 4 = x$ is not continuous at $x = 4$

e) $f(x) = \dfrac{x}{2x^2+x}$ الدالة متصلة إلا عند أصفار المقام

$2x^2 + x = 0 \Rightarrow$ نأخذ عامل مشترك

$\boxed{x = 0}$

$x(2x+1) = 0$ $2x + 1 = 0$: $\boxed{x = -\frac{1}{2}}$

is not continuous at $x = \{-\frac{1}{2}, 0\}$

f) $f(x) \begin{cases} \dfrac{x^2-x}{x^2-1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$

$\boxed{1}$ $f(1) = 1$

$\boxed{2}$ $\lim\limits_{x \to 1} \dfrac{x^2-x}{x^2-1} = \dfrac{1^2-1}{1^2-1} = \dfrac{0}{0}$

$\lim\limits_{x \to 1} \dfrac{x\cancel{(x-1)}}{\cancel{(x-1)}(x+1)} = \lim\limits_{x \to 1} \dfrac{x}{x+1} = \dfrac{1}{1+1} = \dfrac{1}{2}$

is not continuous at $x = 1$

الدالة ليست متصلة عند $x = 1$ لأن $f(1) \neq \lim\limits_{x \to 1}$

Q 8. For what value of the constant k, if possible that will make the function continuous everywhere?

(a). $f(x) = \begin{cases} 7x-2, & if\ x \le 1 \\ kx^2, & if\ x > 1 \end{cases}$ **(b).** $f(x) = \begin{cases} kx^2, & if\ x \le 2 \\ 2x+k, & if\ x > 2 \end{cases}$

(c). $f(x) = \begin{cases} 9-x^2, & if\ x \ge -3 \\ \frac{k}{x^2}, & if\ x < -3 \end{cases}$

ماقيمة k التي تجعل الدالة متصلة :

a) $f(x) \begin{cases} 7x-2 & \text{if } x \le 1 \\ k\,x^2 & \text{if } x > 1 \end{cases}$

$7x - 2 = k\,x^2$

$7(1) - 2 = k(1)^2$

$7 - 2 = k$

$\boxed{5 = k}$ قيمة k التي تجعل الدالة متصلة هي 5

b) $f(x) = \begin{cases} k\,x^2 & \text{if } x \le 2 \\ 2x + k & \text{if } x > 2 \end{cases}$

$k\,x^2 = 2x + k \quad \text{if } x = 2$

$(2)^2 k = 2 \cdot 2 + k$

$4k = 4 + k$

$4k - k = 4$

$3k = 4 \Rightarrow \boxed{k = \frac{4}{3}}$

قيمة k التي تجعل الدالة متصلة عند $x = 2$ هي $\frac{4}{3}$

c) $f(x) \begin{cases} 9 - x^2 & \text{if } x \ge -3 \\ \frac{k}{x^2} & \text{if } x < -3 \end{cases}$

$f(-3) = 9 - (-3)^2 = 9 - 9 = 0$

$\lim_{x \to -3^+} = 9 - (-3)^2 = 9 - 9 = 0$

$\lim_{x \to -3^-} \frac{k}{x^2} = 0 \quad \frac{0}{(-3)^2} = 0$

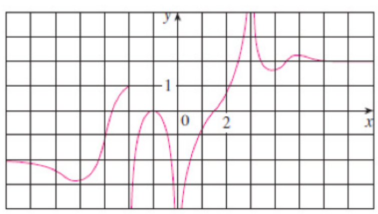
$\therefore k = 0$

القيمة التي تجعل الدالة متصلة عند $x = -3$ هي $k = 0$

Q 9. For the function g whose graph is given, state the following.

(a). $\lim_{x \to \infty} g(x)$ 0 **(b).** $\lim_{x \to -\infty} g(x)$ 0 **(c).** $\lim_{x \to 3} g(x)$ not ecxact

(d). $\lim_{x \to 0} g(x)$ 1 **(e).** $\lim_{x \to -2^+} g(x)$ ∞ **(f).** The equation of the asymptotes 2



Q 10. Find the limits.

(a). $\lim_{x\to\infty}\frac{3x+5}{x-4}$ **(b).** $\lim_{x\to\infty}\frac{\sqrt{9x^6-x}}{x^3+1}$ **(c).** $\lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right)$

(d). $\lim_{x\to\infty}\frac{\sqrt{x^2+2}}{3x-6}$ **(e).** $\lim_{x\to-\infty}\frac{\sqrt{x^2+2}}{3x-6}$ **(f).** $\lim_{x\to\infty}(1+2x-3x^5)$

(g). $\lim_{x\to\infty}\frac{1-e^x}{1+e^x}$ **(h).** $\lim_{x\to-\infty}\frac{1-e^x}{1+e^x}$ **(i).** $\lim_{x\to\infty}\frac{6-t^3}{7t^3+3}$

Q:10 a) $\lim_{x\to\infty}\frac{3x+5}{x-4} = 3$

b) $\lim_{x\to\infty}\frac{\sqrt{9x^6-x}}{x^3+1} = 3$

c) $\lim_{x\to\infty}\sqrt{9x^2+x}-3x = \frac{1}{6}$

d) $\lim_{x\to\infty}\frac{\sqrt{x^2+2}}{3x-6} = \frac{1}{3}$

e) $\lim_{x\to-\infty}\frac{\sqrt{x^2+2}}{3x-6} = -\frac{1}{3}$

f) $\lim_{x\to\infty} 1+2x-3x^5$

$1+2\infty-3\infty = -\infty$

g) $\lim_{x\to\infty}\frac{1-e^x}{1+e^x} = \frac{1-\infty}{1+\infty} = -1$

h) $\lim_{x\to-\infty}\frac{1-e^x}{1+e^x} = \frac{1-0}{1+0} = 1$

i) $\lim_{x\to\infty}\frac{6-t^3}{7t^3+3} = \frac{6-t^3}{7t^3+3}$

Q 11. Find the horizontal and vertical asymptote of the following functions.

(a). $y=\frac{x^2+1}{2x^2-3x-2}$ **(b).** $f(x)=\frac{2x^2+x-1}{x^2+x-2}$

Q 11 a) $y = \frac{x^2+1}{2x^2-3x-2}$

$\lim_{x\to\infty}\frac{x^2+1}{2x^2-3x-2} = \frac{1}{2}$

$\lim_{x\to-\infty}\frac{x^2+1}{2x^2-3x-2} = -\frac{1}{2}$

b) $f(x) = \frac{2x^2+x-1}{x^2+x-2}$

$\lim_{x\to\infty}\frac{2x^2+x-1}{x^2+x-2} = 2$

$\lim_{x\to-\infty}\frac{2x^2+x-1}{x^2+x-2} = -2$