Homework Chapter 3

Q 1. Find the equation of the tangent line to the curve at the specified points.

(a). $y = \dfrac{2x}{x+1}$ at $(1, 1)$

(b). $y = 2x^3 - 5x$ at $(-1, 3)$

(c). $y = 2xe^x$ at $(0, 0)$.

Q 2. Find the derivative of the function $f(x) = x^2 - x$ by using the Ist principle and use to find the equation of tangent line at $x = 2$.

Q 3. Find the derivative of the function $f(x) = x^2 - x$ by using the Ist principle (limit method) and use to find the equation of tangent line at $x = 2$.

Q 4. Find the equation of the tangent line to the curve at the specified points.

(a). $y = \dfrac{2x}{x+1}$ at $(1, 1)$ **(b).** $y = 2x^3 - 5x$ at $(-1, 3)$ **(c).** $y = 2xe^x$ at $(0, 0)$.

Q 5. In each part determine $f'(x)$.

(a). $f(x) = \sqrt{8}$ **(b).** $f(x) = \sqrt[3]{x}$ **(c).** $f(x) = -2x^{-7} + 2\sqrt{x}$

(d). $f(x) = \sqrt{x} - 2e^x$ **(e).** $f(x) = \sqrt{x}(x-1)$ **(f).** $f(x) = \dfrac{x^2 - 2\sqrt{x}}{x}$

Q 6. Find the derivative of the following functions.

(a). $f(x) = (4x^2 - 1)(7x^3 + x)$ **(b).** $f(x) = \dfrac{3x+4}{x^2+1}$ **(c).** $f(x) = \dfrac{x^2 + 4x + 3}{\sqrt{x}}$

(d). $f(x) = (x-1)\,e^x$ **(e).** $f(x) = x^2 . \sin x . \tan x$

(f). $f(x) = (x^2 + 1)\sec x$ **(g).** $f(x) = \dfrac{\sin x . \sec x}{1 + x\tan x}$

Q 7. Suppose that $f(2) = -3$, $g(2) = 4$, $f'(2) = -2$ and $g'(2) = 7$. Find $h'(2)$.

(a). $h(x) = 5f(x) - 4g(x)$ **(b).** $h(x) = f(x).g(x)$

(c). $h(x) = \dfrac{f(x)}{g(x)}$ **(d).** $h(x) = \dfrac{g(x)}{1 + f(x)}$

Q 8. Find the limit.

(a). $\lim\limits_{x \longrightarrow 0} \dfrac{\sin 3x}{x}$ **(b).** $\lim\limits_{x \longrightarrow 0} \dfrac{\sin 5x}{\sin 2x}$

(c). $\lim\limits_{x \longrightarrow 0} \dfrac{\cos x - 1}{\sin x}$ **(d).** $\lim\limits_{x \longrightarrow 0} \dfrac{\sin x}{1 + \tan x}$

Hint: $\left\{ \lim\limits_{x \longrightarrow 0} \dfrac{\sin x}{x} = 1, \quad \lim\limits_{x \longrightarrow 0} \dfrac{\cos x - 1}{x} = \lim\limits_{x \longrightarrow 0} \dfrac{1 - \cos x}{x} = 0 \right\}$

Q 9. Find the derivative of the following functions.

(a). $f(x) = \sqrt[3]{1+\tan x}$ **(b).** $f(x) = e^{x.\cos x}$ **(c).** $f(x) = \left[x + \left(x + \sin^2 x\right)^3\right]^4$

(d). $f(x) = \dfrac{1}{\left(x^4+1\right)^3}$ **(e).** $f(x) = \sin^2\left(e^{\sin x}\right)$ **(f).** $f(x) = x^3.\sin^2(5x)$

(g). $f(x) = \left[1 + \sin^3\left(x^5\right)\right]^{12}$

Q 10. Find $\dfrac{dy}{dx}$ by implicit differentiation.

(a). $x^2y^2 + x\sin y = 4$

(b). $4\cos x.\sin x = 1$

(c). $\sqrt{x+y} = 1 + x^2y^2$

Q 11. Find the derivative of the following functions.

(a). $f(x) = \sin^{-1}\left(\sqrt{\sin\theta}\right)$

(b). $f(x) = \sqrt{1-x^2}.(arc\sin x)$

(c). $f(x) = \sqrt{\tan^{-1} x}$

Q 12. Find the derivative of the following functions.

(a). $f(x) = \ln\left(x^2+10\right)$ **(b).** $f(x) = \log_2(1-3x)$

(c). $f(x) = \ln(\sec x + \tan x)$ **(d).** $y = \left[\ln\left(1+e^x\right)\right]^2$

Q 13. Find y', if $x^y = y^x$. [**Hint:** Taking ln on both side]

Q 14. Use logarithmic differentiation to find the derivative of the following functions.

(a). $y = x^{\cos x}$ **(b).** $y = \sqrt{x^x}$ **(c).** $y = (\sin x)^{\ln x}$

Q 15. Find the derivative of the following functions.

(a). $f(x) = \ln\left(x^2+1\right)$

(b). $y = \ln\left(\dfrac{x^2.\sin x}{\sqrt{1+x}}\right)$

(c). $f(x) = \ln\left[(x-1)^3.(x^2+1)^4\right]$

(d). $y = \left(x^2+1\right)^{\sin x}$

السؤال الأول :- **السؤال الأول**

a) $y = \frac{2x}{x+1}$ at $(1,1)$

المشتقة

$$y' = \frac{(2)(x+1) - (1)(2x)}{(x+1)^2} = \frac{2x+2-2x}{(x+1)^2} = \boxed{\frac{2}{(x+1)^2}}$$

$m = \frac{2}{(1+1)^2} = \frac{2}{(2)^2} = \frac{2}{4} = \boxed{\frac{1}{2}}$ الميل

$y - 1 = \frac{1}{2}(x-1) \Rightarrow y = \frac{1}{2}x - \frac{1}{2} + 1$

$$\boxed{y = \frac{1}{2}x + \frac{1}{2}}$$

b) $y = 2x^3 - 5x$ at $(-1, 3)$

$y' = 6x^2 - 5 \Rightarrow$ المشتقة

$m = 6(-1)^2 - 5 = 6 - 5 = \boxed{1} \Rightarrow$ الميل

$y - 3 = 1(x+1) \Rightarrow y - 3 = x + 1$

$$\boxed{y = x + 4}$$

c) $y = 2xe^x$ at $(0,0)$

ضرب مشتقات :-

1) $2x \longrightarrow 2$
2) $e^x \longrightarrow e^x$

$y' = 2(e^x) + e^x(2x)$

$y' = 2e^x + 2xe^x = \boxed{2e^x(1+x)} \rightarrow$ المشتقة

$m = 2e^0(1+0) = 2 \cdot 1 = \boxed{2} \rightarrow$ الميل

$y - 0 = 2(x-0) = \boxed{y = 2x}$

السؤال الثاني **السؤال الثاني**

2) $f(x) = x^2 - x$ at $x = 2$

$f'(x) = \boxed{2x - 1}$ المشتقة

$m = f'(2) = 2 \cdot 2 - 1 = \boxed{3}$ الميل

$f(2) = y = (2)^2 - 2 = 4 - 2 = \boxed{2}$ قيمة y

$P.(2,2)$ $y - 2 = 3(x-2)$

$y = 3x - 6 + 2$ $\boxed{y = 3x - 4}$

السؤال الثالث

3 $f(x) = x^2 - x$ $\quad x = 2$ [1] نوجد المشتقة بالتعريف

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{[(x+h)^2-(x+h)-[x^2-x]]}{h} = \lim_{h\to 0} \frac{x^2+2hx+h^2-x-h-x^2+x}{h}$$

$$= \lim_{h\to 0} \frac{2hx+h^2-h}{h} = \lim_{h\to 0} \frac{h(2x+h-1)}{h} = \lim_{h\to 0} 2x+h-1 = \boxed{2x-1}$$ المشتقة

$m = f'(2) = 2\cdot 2 - 1 = 4-1 = \boxed{3}$ → الميل

$P = (2, 2)$ $\quad m = 3$

$y - 2 = 3(x-2) \Rightarrow y - 2 = 3x - 6$

$$\boxed{y = 3x - 4}$$

لإيجاد قيمة الـ y .. نعوض في الدالة الأساسية

$y = f(2) = x^2 - x = (2)^2 - 2 = 4-2 = \boxed{2}$ (y)

السؤال الرابع عكس نفس السؤال الأول ..

السؤال الخامس :

5 أوجد مشتقات الدوال

a) $f(x) = \sqrt{8}$ $\quad f'(x) = 0$

b) $f(x) = \sqrt[3]{x}$ $\quad f'(x) = x^{\frac{1}{3}} = \frac{1}{3} x^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{x^2}}$

c) $f(x) = -2x^{-7} + 2\sqrt{x}$

$$f'(x) = +14x^{-8} + \cancel{2}\frac{1}{\cancel{2}\sqrt{x}} = \frac{14}{x^8} + \frac{1}{\sqrt{x}}$$

d) $f(x) = \sqrt{x} - 2e^x$

$$= \frac{1}{2\sqrt{x}} - 2e^x$$

ضرب مشتقات

① $\sqrt{x} \longrightarrow \frac{1}{2\sqrt{x}}$

② $x-1 \longrightarrow 1$

e) $f(x) = \sqrt{x}(x-1)$

$$\frac{x-1}{2\sqrt{x}} + \sqrt{x} = \frac{x-1+\sqrt{x}\cdot 2\sqrt{x}}{2\sqrt{x}}$$

$$= \frac{x-1+2x}{2\sqrt{x}} = \frac{3x-1}{2\sqrt{x}}$$

f) $f(x) = \frac{x^2 - 2\sqrt{x}}{x}$

قسمة مشتقات

[1] $x^2 - 2\sqrt{x} \longrightarrow 2x - \frac{1}{\sqrt{x}}$

[2] $x \longrightarrow 1$

$$f'(x) = \frac{(2x - \frac{1}{\sqrt{x}})(x) - (1)(x^2 - 2\sqrt{x})}{x^2}$$

$$= \frac{2x^2 - \frac{x}{\sqrt{x}} - x^2 - 2\sqrt{x}}{x^2} = \frac{\frac{2x^2\sqrt{x} - x}{\sqrt{x}} - x^2 + 2\sqrt{x}}{x^2}$$

$$= \frac{2x^2\sqrt{x} - x - x^2\sqrt{x} + 2x}{x\sqrt{x}} = \frac{x^2\sqrt{x} + x}{x^2\sqrt{x}} = 1 + \frac{1}{x\sqrt{x}}$$

السؤال السادس :- أوجد المشتقات :-

a $f(x) = (4x^2 - 1)(7x^3 + x)$

ضرب مشتقات

1. $4x^2 - 1 \rightarrow 8x$
2. $7x^3 + x \rightarrow 21x^2 + 1$

$f'(x) = 8x(7x^3 + x) + (4x^2 - 1)(21x^2 + 1)$

$= 56x^4 + 8x^2 + 84x^4 + 4x^2 - 21x^2 - 1$

$= 140x^4 - 9x^2 - 1$

b $f(x) = \frac{3x + 4}{x^2 + 1}$

قسمة مشتقات :

1. $3x + 4 \rightarrow 3$
2. $x^2 + 1 \rightarrow 2x$

$f'(x) \frac{3(x^2+1) - 2x(3x+4)}{(x^2+1)^2}$

$= \frac{3x^2 + 3 - 6x^2 - 8x}{(x^2+1)^2} = \frac{-3x^2 - 8x + 3}{(x^2+1)^2}$

c $f(x) = \frac{x^2 + 4x + 3}{\sqrt{x}}$

قسمة مشتقات

1. $x^2 + 4x + 3 \rightarrow 2x + 4$
2. $\sqrt{x} \rightarrow \frac{1}{2\sqrt{x}}$

$f'(x) \frac{\sqrt{x}(2x+4) - \frac{1}{2\sqrt{x}} \cdot x^2 + 4x + 4}{(\sqrt{x})^2}$

$= \frac{2x\sqrt{x} + 4\sqrt{x} - \frac{x^2 + 4x + 3}{2\sqrt{x}}}{(\sqrt{x})^2} = \frac{3x^2 + 4x - 3}{2x\sqrt{x}}$

d $f(x) = (x - 1)e^x$

ضرب مشتقات

1. $x - 1 \rightarrow 1$
2. $e^x \rightarrow e^x$

$e^x + e^x(x - 1)$

$= e^x + xe^x - e^x = xe^x$

e $f(x) = x^2 \sin x \tan x =$

$2x \sin x \tan x + x^2 \cos x \tan x + x^2 \sin x \sec^2 x$

[f] $f(x) = (x^2+1)\sec x$

ضرب مشتقات

① $x^2+1 \longrightarrow 2x$

② $\sec x \longrightarrow \tan x \cdot \sec x$

$2x\sec x + (x^2+1)\tan x \cdot \sec x$

$\sec x(2x + (x^2+1)\tan x)$

[g] $f(x) = \dfrac{\sin x \sec x}{1 + x\tan x}$

قسمة مشتقات

السبط ضرب مشتقات

$\sin x \longrightarrow \cos x$

$\sec x \longrightarrow \tan x \cdot \sec x$

$\cos x \cdot \sec x + \sin x \cdot \tan x \sec x$

$\sec x(\cos x + \sin x \cdot \tan x)$

القام قسمة مشتقات

$x \longrightarrow 1$

$\tan x \longrightarrow \sec^2 x$

$\tan x + x\sec^2 x$

[1] $\sin x \sec x \longrightarrow \sec x(\cos x + \sin x \tan x)$

[2] $1 + x\tan x \longrightarrow \tan x + x\sec^2 x$

$$\frac{(1+x\tan x)(\sec x(\cos x+\sin x\tan x)) - (\sin x+\sec x)(\tan x + x\sec^2 x)}{(1+x\tan x)^2}$$

السؤال السابع:

[1] $h(x) = 5f(x) - 4g(x)$

$5f'(x) - 4g'(x) \Rightarrow h'(2) = 5f'(2) - 4g'(x)$

$= 5(-2) - 4(7) = -10 - 28 = -38$

[b] $h(x) = f(x)g(x)$

$h'(x) = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

$h'(2) = f'(2)\cdot g(2) + f(2)g'(2)$

$(-2)g(2) + (-3)g'(3)$

$= (-2)\cdot 4 - 3\cdot 7 = -8 - 21 = -29$

c) $h(x) = \frac{f(x)}{g(x)}$ $\quad h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

$h'(2) \frac{(-2)\cdot 4 - (-3)\cdot 7}{(4)^2} = \frac{-8 + 21}{16} = \frac{13}{16}$

d) $h(x) = \frac{g(x)}{1 + f(x)} = \frac{(1 + f(x))\cdot g'(x) - g(x) f'(x)}{(1 + f(x))^2}$

$h(2) \frac{1 + f(2)\cdot g'(2) - g(2)\cdot f'(2)}{(1 + f(2))^2}$

$= \frac{(1 + -3)\cdot 7 - 4(-2)}{(1 + -3)^2} = \frac{-2\cdot 7 - 4\cdot 2}{(-2)^2}$

$= \frac{-14 - 8}{4} = -14 + 2 = -12$

السؤال الثامن

a) $\lim_{x \to 0} \frac{\sin 3x}{x} = \boxed{3}$

b) $\lim_{x \to 0} \frac{\sin 5x}{\sin 2x} = \lim_{x \to 0} \frac{\frac{\sin 5x}{x}}{\frac{\sin 2x}{x}} = \boxed{\frac{5}{2}}$

c) $\lim_{x \to 0} \frac{\cos x - 1}{\sin x} = \lim_{x \to 0} \frac{\frac{\cos x - 1}{x}}{\frac{\sin x}{x}} = \frac{0}{1} = \boxed{0}$

d) $\lim_{x \to 0} \frac{\sin x}{1 + \tan x} = \boxed{0}$

السؤال بتبع :

a $f(x) = \sqrt[3]{1+\tan x} = (1+\tan x)^{\frac{1}{3}}$

$f'(x) = \dfrac{\sec^2 x}{3\sqrt[3]{(1+\tan)^2}}$

الأس ضرب مشتقات

$x \longrightarrow 1$

$\cos x \longrightarrow -\sin x$

b $e^{x\cos x}$

$e^{x\cos x}(-x\sin x + \cos x)$

c $f(x)\left[x + (x+\sin^2 x)^3\right]^4$

$4\left[x+(x\sin^2 x(x))^3\right]\cdot\left(1+3(x+\sin^2 x)^2\cdot(1+2\sin x\cdot\cos x)\right)$

قسمة مشتقات :

$1 \longrightarrow 0$

$(x^4+1)^3 \longrightarrow 3(x^4+1)^2\cdot 4x^3 = 12x^3(x^4+1)^2$

d $f(x) = \dfrac{1}{(x^4+1)^3}$

$f'(x) \dfrac{0\cdot(x^4+1)^3 - 12x^3(x^4+1)^2}{((x^4+1)^3)^2}$

$= \dfrac{-12x^3(x^4+1)^2}{(x^4+1)^6}$

e $f(x) = \sin^2(e^{\sin x})$

$\sin x \quad\quad \sin x$

$\cos x\, e \cdot \sin(2e)$

f $f(x) = x^3\sin^2 5x$

$3x^2\sin^2 5x + 10x^3\cos 5x \sin 5x$

g) $f(x) \left[1+\sin^3(x^5)\right]^{12}$

$12\left[1+\sin^3(x^5)\right]^{11} \cdot (15x^4 \sin^2 x^5 \cos x^5)$

$= 180\, x^4 \sin^2 x^5 \cos x^5 \left[1+\sin^3(x^5)\right]^{11}$

السؤال العاشر:

a) $x^2y^2 + x\sin y = 4$

ضرب مشتقات

$x^2 \longrightarrow 2x$

$y^2 \longrightarrow 2y\frac{dy}{dx}$

ضرب مشتقات

$x \longrightarrow 1$

$\sin y \longrightarrow \cos y \frac{dy}{dx}$

$\frac{d}{dx} x^2y^2 + x\sin y = \frac{d}{dx} 4$

$2xy^2 + 2yx^2\frac{dy}{dx} + \sin y + x\cos y\frac{dy}{dx} = 0$

$2yx^2\frac{dy}{dx} + x\cos y\frac{dy}{dx} = -2xy^2 - \sin y$

$(2yx^2 + x\cos y)\frac{dy}{dx} = -(2xy^2 + \sin y)$

$\frac{dy}{dx} = \frac{-(2xy^2 + \sin y)}{2yx^2 + x\cos y}$

b) $4\cos x \cdot \sin x = 1$

$\frac{dy}{dx}$ = المشتقة غير موجودة

c) $\sqrt{x+y} = 1 + x^2y^2$

$x^2 \longrightarrow 2x$

$y^2 \longrightarrow 2y\frac{dy}{dx}$

$\frac{1+\frac{dy}{dx}}{2\sqrt{x+y}} = 0 + \left(2xy^2 + 2x^2y\frac{dy}{dx}\right) 2\sqrt{x+y}$

$1 + \frac{dy}{dx} = 4xy^2\sqrt{x+y} + 4x^2y\sqrt{x+y}\frac{dy}{dx}$

$$\frac{dy}{dx} - 4x^2y\sqrt{x+y}\,\frac{dy}{dx} = -1 + 4xy^2\sqrt{x+y}$$

$$(1 - 4x^2y\sqrt{x+y})\,\frac{dy}{dx} = 4xy^2\sqrt{x+y} - 1$$

$$\frac{dy}{dx} = \frac{4xy^2\sqrt{x+y} - 1}{1 - 4x^2y\sqrt{x+y}}$$

السؤال الحادي عشر:

[a] $f(x) = \sin^{-1}(\sqrt{\sin\theta})$

$$\frac{\cos\theta}{2\sqrt{\sin\theta}\cdot\sqrt{1-\sin\theta}}$$

[b] $f(x) = \sqrt{1-x^2}\cdot \arcsin x$

$$\frac{-x}{\sqrt{1-x^2}}\cdot \arcsin + \frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}$$

$$= \frac{-x\arcsin x}{\sqrt{1-x^2}} + 1$$

ضرب مشتقات

$\sqrt{1-x^2} \rightarrow \frac{-x}{\sqrt{1-x^2}}$

$\arcsin \rightarrow \frac{1}{\sqrt{1-x^2}}$

[c] $f(x) = \sqrt{\tan^{-1}x}$

$$\frac{1}{2\sqrt{\tan^{-1}x}\,(x^2+1)}$$

السؤال الثاني عشر

[a] $f(x) = \ln(x^2+10)$

$$f'(x) = \frac{2x}{x^2+10}$$

[b] $f(x) = \log_2(1-3x)$

$$f'(x) \quad \frac{-3}{(1-3x)\ln 2}$$

c) $f(x) = \ln(\sec x + \tan x)$

$$f'(x) = \frac{\tan x \cdot \sec x + \sec^2 x}{\sec x + \tan x} = \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x}$$

d) $y = [\ln(1+e^x)]^2$

$$y' = 2[\ln(1+e^x)] \cdot \frac{e^x}{1+e^x}$$

$x^y = y^x$ find y'

السؤال إلت فات شرح

1 ندخل لن لـ الطرفين

$$\ln x^y = \ln y^x$$

$$y \ln x = x \ln y$$ ← 2 ندخل خصائص لن

$$\frac{d}{dx} y \ln x = \frac{d}{dx} x \ln y$$ ← 3 نشتق الطرفين

$$\frac{dy}{dx} \ln x + \frac{y}{x} = \ln y + \frac{x}{y} \frac{dy}{dx}$$

$$\left(\ln x - \frac{x}{y}\right) \frac{dy}{dx} = \ln y - \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{\ln y - \frac{y}{x}}{\ln x - \frac{x}{y}}$$

$$\frac{dy}{dx} = \frac{-\ln y + \frac{y}{x}}{\ln x + \frac{x}{y}}$$

مترب مشتقات

$y \rightarrow \frac{dy}{dx}$

$\ln x \rightarrow \frac{1}{x}$

$x \rightarrow 1$

$\ln y \rightarrow \frac{1}{y}\frac{dy}{dx}$

السؤال الرابع عشر

[a] $y = x^{\cos x}$

$\ln y = \ln x^{\cos x}$

نضيف ln الطرفين

نستخدم خصائص ln

$\ln y = \cos x \ln x$

نشتق الطرفين

$\frac{1}{y}\frac{dy}{dx} = -\sin x \ln x + \frac{1}{x}\cos x$

$\frac{dy}{dx} = y\left(-\sin x \ln x + \frac{1}{x}\cos x\right)$

$\frac{dy}{dx} = x^{\cos x}\left(-\sin x \ln x + \frac{1}{x}\cos x\right)$

[b] $y = \sqrt{x^x}$ $\quad y = (x^x)^{\frac{1}{2}}$

نحول الجذر إلى أس

$\ln y = \ln\left(x^{\frac{1}{2}x}\right)$

نضيف ln الطرفين

$\ln y = \frac{1}{2}x \ln x$

نستخدم خصائص ln

$y' \cdot \frac{1}{y} = \frac{1}{2}\ln x + \frac{1}{2x}\cancel{x}$

نشتق الطرفين

$y' = y\left(\frac{1}{2}\ln x + \frac{1}{2}\right) \quad = y' = \sqrt{x^x}\left(\frac{1}{2}\ln x + \frac{1}{2}\right)$

[c] $y = (\sin x)^{\ln x}$

نضيف ln الطرفين

$\ln y = \ln (\sin x)^{\ln x}$

$\ln y = \ln x \cdot \ln \sin x$

نستخدم خصائص ln

$y' \cdot \frac{1}{y} = \frac{1}{x}\ln \sin x + \ln x \frac{\cos x}{\sin x}$

نشتق الطرفين

$y' = (\sin x)^{\ln x}\left[\frac{1}{x}\ln \sin x + \cot x \ln x\right]$

السؤال الخامس عشر

a) $f(x) = \ln(x^2+1)$

$f'(x) = \frac{2x}{x^2+1}$

b) $y = \ln\left(\frac{x^2 \cdot \sin x}{\sqrt{1+x}}\right)$ باستخدام خصائص ln نرتب لدالة

$y = \ln x^2 \cdot \sin x - \ln(1+x)^{\frac{1}{2}}$

$y = \ln x^2 \cdot \sin x - \frac{1}{2}\ln(1+x)$

$x^2 \longrightarrow 2x$
$\sin x \longrightarrow \cos x$

$y' = \frac{2x\sin x + x^2\cos x}{x^2\sin x} - \frac{1}{2}\cdot\frac{1}{1+x}$

$y' = \frac{2x\sin x}{x^2\sin x} + \frac{x^2\cos x}{x^2\sin x} - \frac{1}{2(1+x)}$

$y' = \frac{2}{x} + \cot x - \frac{1}{2(1+x)}$

c) $f(x) = \ln\left[(x-1)^3 \cdot (x^2+1)^4\right]$ نستخدم خصائص ln ونرتب المسألة

$f(x) = \ln(x-1)^3 + \ln(x^2+1)^4$

$3\ln(x-1) + 4\ln(x^2+1)$ نشتق

$f'(x) = \frac{3}{x-1} + \frac{4\cdot 2x}{x^2+1} = \frac{3}{x-1} + \frac{8x}{x^2+1}$

d) $y = (x^2+1)^{\sin x}$ نضيف ln للطرفين

$\ln y = \ln(x^2+1)^{\sin x}$ نستخدم خصائص ln

$\ln y = \sin x \ln(x^2+1)$ نشتق

$$y' \cdot \frac{1}{y} = \cos x \ln(x^2+1) + \sin x \frac{2x}{x^2+1}$$

$$y' = y\left[\cos x \ln(x^2+1) + \sin x \frac{2}{x^2+1}\right]$$

$$y' = (x^2+1)^{\sin x}\left[\cos x \ln(x^2+1) + \sin x \frac{2}{x^2+1}\right]$$