Assignment 3: Equations, Inequalities, Functions, Matrices and Statistics

1) **Solve the following equations:**

a) $4x + 3 = 5x - 7$ b) $\frac{3x+4}{x-2} = \frac{3}{7}$ c) $3(x - 2) = -4(x + 6)$

d) $5 - \frac{3a+6}{4} = \frac{6-7a}{2}$

e) $3x^2 - 2x - 1 = 0$

f) $(x + 4)(x - 2) = 0$ g) $2x^2 = 5x$

h) $(2x - 4)^2 = 5$ (Use: if $A^2 = c$ then $A = \pm\sqrt{c}$)

2) **Rewrite each of the following in inequality notation and graph on a real number line:**

a) $[-2, 3)$ b) $(-\infty, 3)$ c) $[4, +\infty)$ d) $(-2, 3]$

3) **Rewrite each of the following in interval notation and graph on a real number line:**

a) $-4 < x \leq 7$ b) $x < 4$ c) $x \geq -5$

4) **Solve and graph:**

a) $-2x > 4$ b) $-2x+3 < 7$

5) **Determine whether each set specifies a function. If it does, then state the domain and the range**

a) S= {(1,4), (2,3), (3,2), (4,3), (5,4)}

b) K= {(1,4), (1,3), (3,2), (4,3), (5,4)}

6) **Find** $g(1), 2g(h)$, *and* $g(1 + h)$ *for* $g(x) = 16 + 3x - x^2$

7) **Find the domain of the following functions:**

a) $f_1(x) = 3 + x$ b) $f_2(x) = \sqrt{3 + x}$

c) $f_3(x) = \frac{1}{3+x}$ d) $f_4(x) = \sqrt[5]{3 + x}$

8) **Determine which of the following functions is even, odd, or neither**

a) $f_1(x) = x^7 + x$ b) $f_2(x) = x^4 - 3x^2$ c) $f_3(x) = x^7 + 3$

9) ***For*** $f(x) = 3x^2 + 1$ ***and*** $g(x) = -2x + 3$. ***Find the functions:***

a) $(f + g)(x)$ b) $(f - g)(x)$ c) $(fg)(x)$ d) $(f/g)(x)$

e) $fog(x)$ f) $gof(x)$ ***and find their domains.***

10) *Find* a, and b such *that* $3\begin{pmatrix} a & 1 \\ 0 & 2 \end{pmatrix} - 4\begin{pmatrix} 5 & b \\ 5 & 4 \end{pmatrix} = \begin{pmatrix} 10 & 8 \\ -20 & -10 \end{pmatrix}$

11) Perform the indicated operation if it's possible:

السؤال الأول

1

a) $4x + 3 = 5x - 7$

$3 + 7 = 5x - 4x$

$10 = x$

b) $\frac{3x+4}{x-2} = \frac{3}{7}$

$7(3x+4) = 3(x-2)$

$21x + 28 = 3x - 6$

$21x - 3x = -6 - 28$

$\frac{18}{18}x = \frac{34}{18}$ $\div 2$ $\div 2$ $\quad -\frac{17}{9}$

$x = -\frac{17}{9}$

c) $3(x-2) = -4(x+6)$

$3x - 6 = -4x - 24$

$3x + 4x = -24 + 6$

$\frac{7}{7}x = \frac{-18}{7}$

$x = -\frac{18}{7}$

d) $8(5) - 8\left(\frac{3a+6}{4}\right)^2 = \left(\frac{6-7a}{2}\right)^4 \times 8$

$40 - 2(3a+6) = 4(6-7a)$

$40 - 6a - 12 = 24 - 28a$

$-6a + 28 = 24 - 28a$

$-6a + 28a = 24 - 28$

$\frac{22}{22}a = -\frac{4}{22}$ ÷2 ÷2 $\frac{2}{11}$

$a = -\frac{2}{11}$

e) $3x^2 - 2x - 1 = 0$ (3x, x, -1, +1)

$(3x+1)(x-1) = 0$

الحل بالتحليل بأقواس

$3x+1=0$ | $x-1=0$

$\frac{3}{3}x = \frac{-1}{3}$ | $x = 1$

$x = -\frac{1}{3}$ | $x = 1$

الحل بالقانون العام والتحليل بأقواس

$x = \{-\frac{1}{3}, 1\}$

f) $(x+4)(x-2)=0$

$x+4=0$ | $x-2=0$

$x=-4$ | $x=+2$

$x\{-4,+2\}$

g) $2x^2=5x$

$2x^2-5x=0$ ← ناخذ x عامل مشترك

$x(2x-5)=0$

x تساوي صفر — $2x-5$ تساوي صفر

$x=\{0,\frac{5}{2}\}$

$x=0$

$2x-5=0$

$\frac{2}{2}x=\frac{5}{2}$

$x=\frac{5}{2}$

h) $(2x-4)^2=5$ ← $(2x-4)^2$ الحد الأول داخل القوس مربع تام

$(2x)^2-2(2x)(4)+(4)^2=5$

$4x^2-16x+16-5=5$ ← نحولها معادلة تربيعية ونساويها بالصفر

$4x^2-16x+11=0$

$a = 4 \quad b = -26 \quad c = 11$

لـ حل المعادلة بالقانون العام

$$\frac{-b \pm \sqrt{b^2 - 4a \cdot c}}{2a}$$

$$\frac{-(-26) \pm \sqrt{(-26)^2 - 4 \cdot 4 \cdot 11}}{2 \cdot 4}$$

$$\sqrt{(-26)^2 - 4 \cdot 4 \cdot 11}$$

$$= \sqrt{256 - 176}$$

$$\sqrt{80}$$

$$\sqrt{16}\ \sqrt{5}$$

$$4\sqrt{5}$$

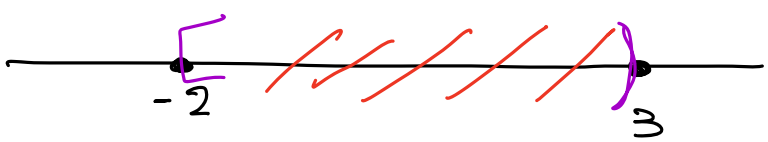
$$+ \frac{26 \pm 4\sqrt{5}}{8}$$

$$\frac{2 \pm \frac{1}{4}\sqrt{5}}{}$$

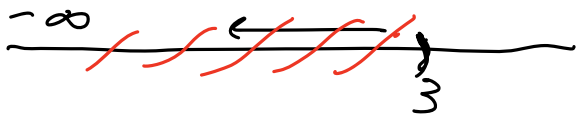
$$x \left\{ 2 + \frac{\sqrt{5}}{2}, \ 2 - \frac{\sqrt{5}}{2} \right\}$$

2 بـــؤال الـثـانـي

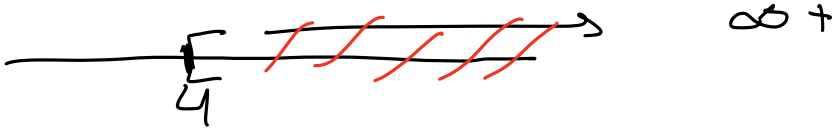
a) $[-2, 3)$ $\quad -2 \leq x < 3$



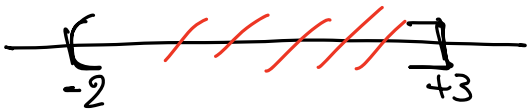
b) $(-\infty, 3)$ $\quad x < 3$



c) $[4, \infty+)$ $\quad 4 \leq x$

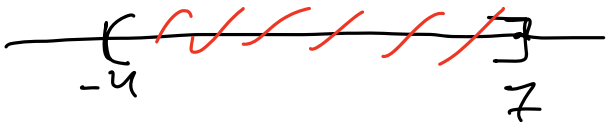


d) $(-2, 3]$ $\quad -2 < x \leq 3$

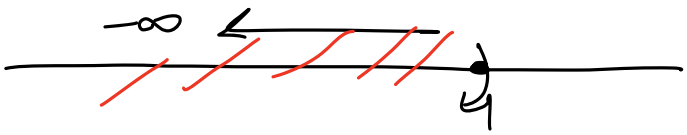


3 السؤال الثالث

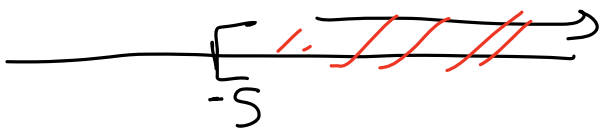
a) $-4 < x \leq 7$ $\quad (-4, 7]$



b) $x < 4$ $\quad (-\infty, 4)$



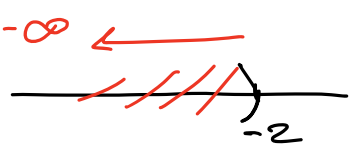
c) $x \geq -5$ $\quad [-5, \infty+)$



4 السؤال الرابع :-

a) $\frac{-2x}{-2} > \frac{4}{-2}$

$x < -2$ $\quad (-\infty, -2)$



b) $-2x+3 < 7-3$

$$\frac{-2x}{-2} < \frac{4}{-2}$$

$$x > -2$$

$(-2, \infty+)$

5 السؤال الخامس

1) $S = \{(1,4), (2,3), (3,2), (4,3), (5,4)\}$

is function

Domain $= \{1, 2, 3, 4, 5\}$

Range $= \{4, 3, 2\}$

2) $K = \{(1,4), (1,3), (3,2), (4,3), (5,4)\}$

is not function

6 السؤال السادس

$g(x) = 16 + 3x - x^2$

$g(1) = 16 + 3(1) - (1)^2 =$

$16 + 3 - 1 = 18$

$2\,g(h)$ $\quad 2(16 + 3h - (h)^2)$

$38 + 6h - 2h^2$

$g(1+h)$ $\quad 16 + 3(1+h) - (1+h)^2$

$$16 + 3 + 3h - 1 - 2h - h^2$$

$$18 + h - h^2$$

(7) الدومين Domain

a) $f_1(x) = 3 + x$

$\mathbb{R}$, $(-\infty, \infty+)$

b) $f_2(x) = \sqrt{3 + x}$

$3 + x \geq 0$

$x \geq -3$ $\quad [-3, \infty+)$

c) $f_3(x)$ $\quad \frac{1}{3+x}$

$3 + x = 0$ $\quad x = -3$

$\mathbb{R} - \{-3\}$ $\quad (-\infty, -3) \cup (-3, \infty+)$

d) $f_4(x)$ $\quad \sqrt[5]{3 + x}$

$(-\infty, \infty+)$ $\quad = \mathbb{R}$

(8) الزوجي الفردي

a) $f_1(x) = x^7 + x$ $\quad$ even

$f(x) \neq f(-x)$ $\quad f(-x) = -f(x)$

b) $f_2(x) = x^4 - 3x^2$ $\quad$ odd

$f(x) = f(-x)$

c) $f_2(x) = x^7 + 3$ neither

$f(x) \neq f(-x) = -f(x)$

السؤال الرابع

9

$f(x) = 3x^2 + 1 \quad g(x) = -2x + 3$

a) $f+g = (3x^2+1) + (-2x+3) = 3x^2 - 2x + 4$

$D = \mathbb{R} \quad (-\infty, \infty)$

b) $f-g = (3x^2+1) - (-2x+3) = 3x^2 + 2x - 2$

$D = \mathbb{R}, \quad (-\infty, \infty)$

c) $fg = (3x^2+1)(-2x+3)$

$3x^2(-2x+3) + 1(-2x+3)$

$-6x^3 + 9x^2 - 2x + 3$

$D = \mathbb{R} \quad (-\infty, \infty)$

d) $\frac{f}{g} = \frac{3x^2+1}{-2x+3}$

$D = -2x + 3 = 0$

$\frac{-2x}{-2} = \frac{-3}{-2} \qquad x = \frac{3}{2}$

$D = \mathbb{R} - \{\frac{3}{2}\}, \quad (-\infty, \frac{3}{2}) \cup (\frac{3}{2}, \infty)$

e) $f \circ g = 3(-2x+3)^2 + 1$

$3(4x^2 - 12x + 9) + 1$

$12x^2 - 36x + 28$

$D = \mathbb{R} \quad (-\infty, \infty+)$

f) $g \circ f = -2(3x^2 + 1) + 3$

$-6x^2 + 1$

$D = \mathbb{R} = (-\infty, \infty+)$

10

السؤال العاشر

find a, b

$$\begin{pmatrix} 3a & 3 \\ 0 & 6 \end{pmatrix} - \begin{pmatrix} 20 & 4b \\ 20 & 16 \end{pmatrix} = \begin{pmatrix} 10 & 8 \\ -20 & -10 \end{pmatrix}$$

$3a - 20 = 10$

$3a = 10 + 20$

$\frac{3a}{3} = \frac{30}{3}$

$a = 10$

$3 - 4b = 8$

$-4b = 8 - 3$

$\frac{-4b}{-4} = \frac{5}{-4}$

$b = -\frac{5}{4}$

a) $(1 \quad 2 \quad -1)\begin{pmatrix} 4 & 5 \\ 2 & -2 \\ 0 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 4 & 5 \\ 2 & -2 \\ 0 & 1 \end{pmatrix}(1 \quad 2 \quad -1)$

c) $\begin{pmatrix} 1 & -1 \\ 5 & 3 \end{pmatrix}\begin{pmatrix} 4 & 2 \\ 8 & 0 \end{pmatrix}$ d) $(1 \quad 2 \quad -1) + \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

e) $3\begin{pmatrix} 1 & 0 & -2 \\ 5 & 7 & 8 \end{pmatrix} - 2\begin{pmatrix} 0 & 4 & 8 \\ 1 & 5 & -5 \end{pmatrix}$

12) If $A = \begin{pmatrix} 1 & 0 \\ 0 & 7 \end{pmatrix}$. Calculate $A^2 - 3A + 5I$

(Hint: $A^2 = A\,A$, $A^3 = A\,A\,A$)

13) Find the inverse of the following matrices:

a) $A = \begin{pmatrix} 1 & 3 \\ -5 & 7 \end{pmatrix}$ b) $B = \begin{pmatrix} 2 & 4 \\ 2 & 4 \end{pmatrix}$

14)Twenty-five army inductees were given a blood test to determine their blood type. The data set is A B AB AB O O O B AB AB B B O A O A O O O AB AB A O B AB

a) Construct a frequency distribution for the data
b) Find the percentages for each class
c) Draw the pie graph
d) Draw the bar graph

15) These data represent the record high temperatures in degrees Fahrenheit (%F) for each of the 50 states.

a) Construct a grouped frequency distribution and a cumulative frequency distribution for the data using 5 classes.

112 100 127 120 134 118 105 110 109 112 110 118 117 116
118 122 114 114 105 109 107 112 114 115 118 117 118 122
106 110 116 108 110 121 113 120 119 111 104 111 120 113
120 117 105 110 118 112 114 116

b) Draw the histogram, frequency polygon and Ogive for the above data.

16) for the following data *20, 26, 40, 36, 23, 42, 35, 24, 30, 36*
find the Mean, Median, mode, Range, and Midrange.

(11)

إيجاد السؤال الحادي عشر

a) $\begin{pmatrix} 1 & 2 & -1 \end{pmatrix}_{1,3} \begin{pmatrix} 4 & 5 \\ 2 & -2 \\ 0 & 1 \end{pmatrix}_{3,2}$

$\begin{pmatrix} 4+4+0 & 5+(-4)+(-1) \end{pmatrix} = \begin{pmatrix} 8 & 0 \end{pmatrix}$

b) $\begin{pmatrix} 4 & 5 \\ 2 & -2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & -1 \end{pmatrix}$

* عملية الضرب مستحيلة
لا تحقق الشرط

c) $\begin{pmatrix} 1 & -1 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} 4 & 2 \\ 8 & 0 \end{pmatrix}$

$\begin{pmatrix} 4+(-8) & 2+0 \\ 20+24 & 10+0 \end{pmatrix} = \begin{pmatrix} -4 & 2 \\ 44 & 10 \end{pmatrix}$

d) $\begin{pmatrix} 1 & 2 & -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

لا يمكن الجمع لا تحقق الشرط

e) $3\begin{pmatrix} 1 & 0 & -2 \\ 5 & 7 & 8 \end{pmatrix} - 2\begin{pmatrix} 0 & 4 & 8 \\ 1 & 5 & -5 \end{pmatrix}$

$$\begin{pmatrix} 3 & 0 & -6 \\ 15 & 21 & 24 \end{pmatrix} + \begin{pmatrix} -0 & -8 & -16 \\ -2 & -20 & +20 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & -8 & -22 \\ 13 & 11 & 34 \end{pmatrix}$$

(12) السؤال الثاني عشر

$A^2 - 3A + 5I$

$A = \begin{pmatrix} 1 & 0 \\ 0 & 7 \end{pmatrix}$ ① $A^2 = A \cdot A$

$$\begin{pmatrix} 1 & 0 \\ 0 & 7 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 49 \end{pmatrix}$$

② $3\begin{pmatrix} 1 & 0 \\ 0 & 7 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 0 & 21 \end{pmatrix}$

③ $A^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 7 \end{pmatrix}$ $7 - 0 = 7$

$$A^{-1} = \begin{pmatrix} \frac{1}{7} & \frac{0}{7} \\ \frac{0}{7} & \frac{7}{7} \end{pmatrix} = \begin{pmatrix} \frac{1}{7} & 0 \\ 0 & 1 \end{pmatrix}$$

$$I = A \cdot A^{-1} = \begin{pmatrix} \frac{1}{7} & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 7 \end{pmatrix} = \begin{pmatrix} \frac{1}{7} & 0 \\ 0 & 7 \end{pmatrix}$$

$$5I = 5 \begin{pmatrix} \frac{1}{7} & 0 \\ 0 & 7 \end{pmatrix} = \begin{pmatrix} \frac{5}{7} & 0 \\ 0 & 35 \end{pmatrix}$$

$$A^2 - 3A + 5I$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 49 \end{pmatrix} - \begin{pmatrix} 3 & 0 \\ 0 & 21 \end{pmatrix} + \begin{pmatrix} \frac{5}{7} & 0 \\ 0 & 35 \end{pmatrix}$$

$$\begin{pmatrix} -3 & 0 \\ 0 & 38 \end{pmatrix} + \begin{pmatrix} \frac{5}{7} & 0 \\ 0 & 35 \end{pmatrix} = \begin{pmatrix} \frac{26}{7} & 0 \\ 0 & 73 \end{pmatrix}$$

13

السؤال الثالث عشر

$$A = \begin{pmatrix} 1 & 3 \\ -5 & 7 \end{pmatrix} \qquad (1 \cdot 7) - (3 \cdot (-5)) = 7 + 15 = 22$$

$$\text{invers} = \begin{pmatrix} \frac{1}{22} & \frac{3}{22} \\ \frac{-5}{22} & \frac{7}{22} \end{pmatrix}$$

$$B = \begin{pmatrix} 2 & 4 \\ 2 & 4 \end{pmatrix} = (2 \cdot 4) - (2 \cdot 4) = 8 - 8 = 0$$

لا يوجد معكوس لهذه المصفوفة

[14]

السؤال الرابع عشر

A B AB AB O O O B AB AB B B
O A O A O O O AB AB A O B AB

a) نوزع البيانات

A	4
B	5
O	9
AB	7
Total	25

b) النسب المئوية

$\frac{4}{25} \times 100 = 16$

$\frac{5}{25} \times 100 = 20$

$\frac{9}{25} \times 100 = 36$

$\frac{7}{25} \times 100 = 28$

total = 100

c) الرسم البياني

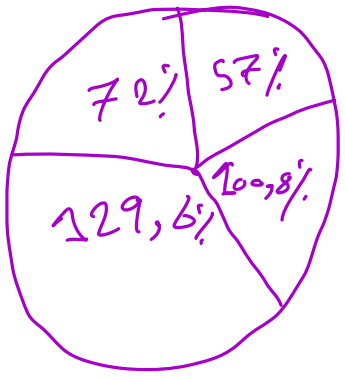
الرسم البياني يعتمد على القيم وحدوية في 360

$A = \frac{4}{25} \times 360 = 57\%$

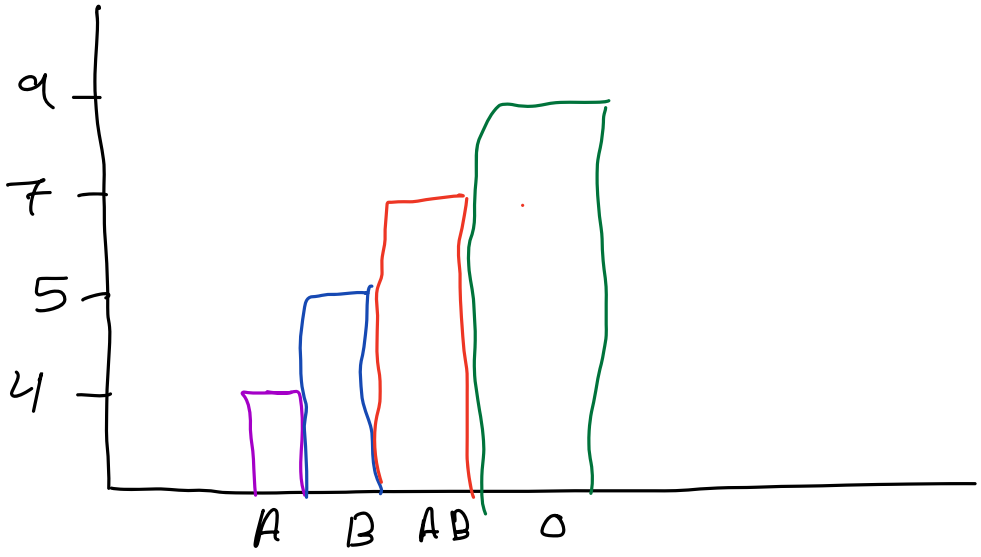
$B = \frac{5}{25} \times 360 = 72\%$

$O = \frac{9}{25} \times 360 = 129,6\%$

$$AB = \frac{7}{25} \times 360 = 100,8°$$



d) ارسم المدرج التكراري



15

السؤال الخامس عشر

a) توزيع البيانات

class limits	class boundaries	frequency
100 - 104	99,5 - 104,5	2
105 - 109	104,5 - 109,5	8

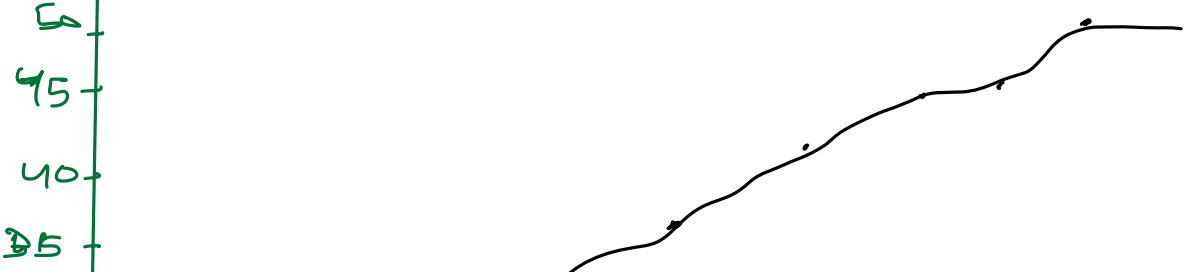
210 - 214	209,5 - 214,5	17
215 - 219	214,5 - 219,5	14
120 - 124	219,5 - 124,5	7
125 - 129	124,5 - 129,5	1
130 - 134	129,5 - 134,5	1

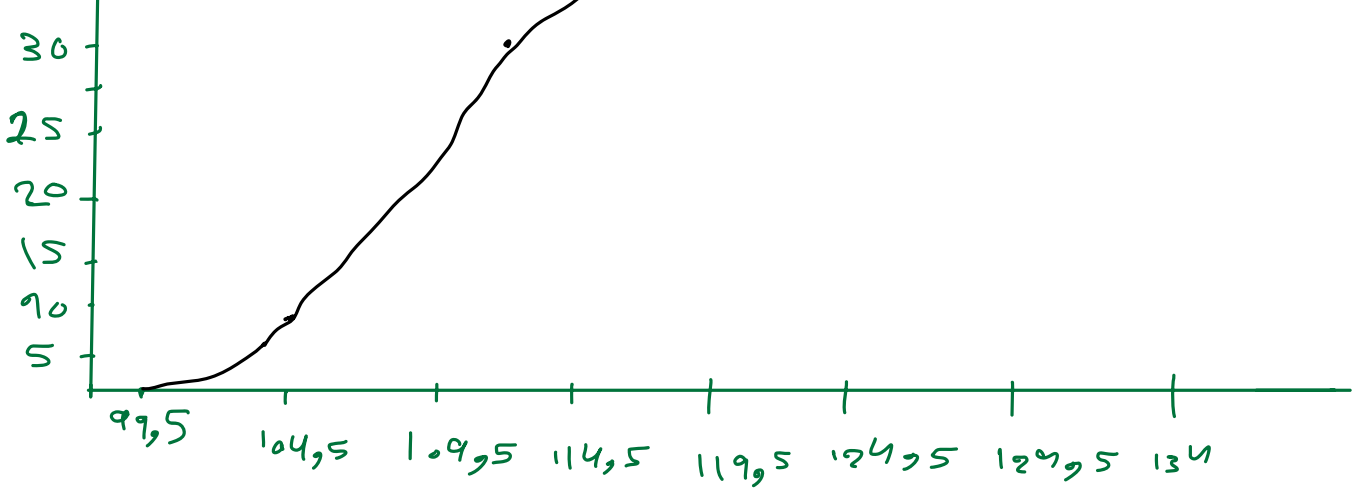
$n = \Sigma f = 50$

cumulative frequency

liss than	99,5	0
liss than	104,5	2
Liss than	209,5	10
Liss than	114,5	27
Liss than	119,5	41
Liss than	124,5	48
Liss than	129,5	49
Liss than	134,5	50

b) البيانية





16 الحل الصحيح

20, 26, 40, 36, 23, 42, 35, 24, 30, 36

a) The Mean $\frac{20+26+40+36+23+42+35+24+30+36}{10}$

$\frac{312}{10} = 31,2$

b) Median 20, 23, 24, 26, 30, 35, 36, 36, 40, 42

$\frac{30+35}{2} = \frac{65}{2} = 32,5$

c) Mode: 36

d) Modren = $\frac{42 + 20}{2} = \frac{62}{2} = 31$